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WADC TECHNICAL REPORT 53-189

STABILITY OF FLOW IN AIR-INDUCTION SYSTEMS FOR BOUNDARY-LAYER SUCTION

By

A. H. Flax

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J. T. Curtis

CORNELL AERONAUTICAL LABORATORY, INC.
Buffalo 21, New York

May 1953

WRIGHT AIR DEVELOPMENT CENTER

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Aeronautical Research Laboratory

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Wright Air Development Center

Air Research and Development Command

United States Air Force

Wright-Patterson Air Force Base, Ohio

FOREWORD

The work described in this report was carried out by the Aerodynamic Research Department of the Cornell Aeronautical Laboratory under Air Force Contract No. AF 33(038)-19242, Expenditure Order No. 460-31-12-15 SR 1g. This contract was administered by the Aeronautical Research Laboratory, Wright Air Development Center, under the technical cognizance of Mr. Lee S. Wasserman.

The experimental work carried out under this contract was supervised by Mr. John G. Wilder of the Cornell Aeronautical Laboratory

ABSTRACT

Theoretical and experimental studies of the stability of flow in air induction systems with high pressure recoveries for boundary-layer suction have been carried out. The boundary-layer flow entering such systems may have the characteristic of increasing its total head with increasing flow rate for part of the operating range. This characteristic produces both static and dynamic instability. The static instability is evidenced by the appearance of unequal flows in ostensibly identical branches of a system. The dynamic instability occurs in the form of regular oscillations of the flow. These oscillations take place at the characteristic frequencies of the duct and plenum chamber system. In simple cases, the measured frequency shows good agreement with that calculated on the basis of the acoustic theory of the Helmholtz resonator. In the experimental investigation, it was shown that the installation of splitter plates in a system with a wide slot which exhibited both static and dynamic instability eliminated the latter but not the former. Both types of instability could be eliminated by introduction of high slot and duct losses into the system or by ingesting a flow quantity into the system sufficient to remove almost the entire boundary layer ahead of all slots. It should be pointed out that a model setup simulating an airfoil boundary layer control system would not exhibit either of the instabilities. This was thought to be possible due to a plenum volume scale effect and/or to the relatively higher losses caused by the model scale effect.

PUBLICATION REVIEW

This report has been reviewed and is approved.

FOR THE COMMANDING GENERAL:

Harry C. Henry Lt Col

for LESLIE B. WILLIAMS, Colonel, USAF
Chief, Aeronautical Research Laboratory
Directorate of Research

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INTRODUCTION

Instability of flow in air induction systems which take in boundary-layer air has often been observed, mainly in connection with engine air intakes. In such cases, the ingestion of boundary-layer air is an incidental and, in most cases, undesirable feature of the system. It has often been found possible to cure the instability, and at the same time to improve the efficiency of the intake by diverting the boundary-layer air through a seperate boundary-layer bleed system. In the case of boundary-layer control systems, however, practically the entire mass of air being sucked through the system is boundary-layer air and the problem of stability of the system needs more careful study. Although the use of boundary-layer suction to improve the lift and drag characteristics of airfoils is an old concept, it has recently received renewed attention (refs. 1,2,3 and 4). In particular, the possibility of stabilization of the laminar boundary layer by suction appears attractive and is being vigorously pursued. Although there is now a very considerable literature dealing with boundary-layer suction, the detailed consideration of the stability of flow in the induction system has been almost entirely ignored. Smith and Roberts (ref. 3) have, however, pointed out that there is a possibility of instability in boundary-layer slots due to the fact that, for small flow rates into the slot, the total head of the air entering the system exhibits a positive increase with increase of volume flow. It is well known that such a flow characteristic is associated with the phenomena of compressor surge (refs. 5 and 6) and supersonic diffuser "buzz". As will be seen later, this characteristic of the slot intake is a necessary but not sufficient condition for instability. Other necessary conditions for instability arise from the characteristics of the duct system. Further, the instability encountered may be either static or dynamic in nature; the type is also determined by the characteristics of the entire induction system.

Smith and Roberts (ref. 3) determined the curve of total pressure recovery versus rate of flow for the flow of boundary layer air into a slot, including the estimated effects of slot losses, which are stabilizing. Their results showed that the slope of the total pressure versus rate-of-flow curve could be positive for the lower rates of flow, which could be typical of those employed for stabilization of the laminar boundary layer. Although the type of instability with which they were concerned is not clear from their paper, Smith, in a private communication to a member of the C.A.L. staff (ref. 11) stated that the investigation was concerned with the problem of long slots which often "show a great rush of air in one side and very slow, or even reverse, flow on the other side". As will be seen later, serious effects of this type do occur, but they may also be accompanied by a resonant oscillation of the system as well.

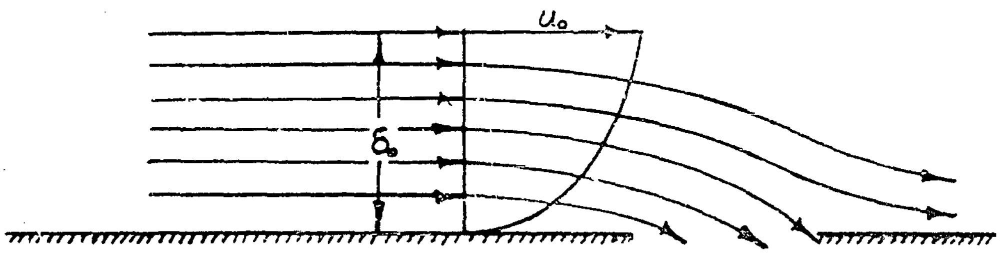
The theoretical analyses presented in this report show that the physical factors underlying the static instability discussed above may also give rise to a dynamic oscillatory instability. It is shown that the positive slope of the curve of total head versus flow rate for the intake slot is equivalent to negative damping imposed on the resonant system made up of the air in the duct system. The resonant character of this system is very much like that of the classical Helmholtz resonator in acoustics (ref. 8). In fact, it was found possible to calculate the resonant frequencies for the cases tested with fair accuracy based on acoustic theory.

As far as was known, there were no detailed experimental data on either the static or dynamic stability of boundary-layer air induction systems available in the literature. Since the details of the flow into a slot and particularly the losses under dynamic conditions cannot be predicted theoretically with satisfactory accuracy, it was difficult to assess the probability of encountering the various types of instability prior to conducting the present experimental program. For this reason, much of the present experimental work was exploratory in nature with the aim of establishing the essential features of the instabilities encountered and the main conditions for their occurrence. Also, the models employed were idealised representations of boundary-layer control systems rather than representative of any practical case.

In the original plan for the present program, it was intended that the investigation be centered on the question of static instability, with only secondary attention to the dynamic problem, since the existence and range of occurrence of the latter were uncertain. Actually, however, the dynamic instability presented itself in the very first tests, and it was necessary to give primary attention to it to the detriment of the static investigations. Pure static instability was observed in the present program only with multiple air intakes, although a non-uniform distribution of the mean flow, of the type described by Smith (ref.11), was also present during resonant oscillation of the system with long narrow slots.

In the original program, it was planned that most of the work would be done with laminar boundary layers representative of boundary-layer control for drag reduction. It turned out, however, that with the small thickness of natural laminar boundary layer available on an airfoil in these experiments, the losses in the very small slots required may have produced a stable total head versus flow rate curve, i.e., no increase in total head with increasing flow rate. Thus, neither static nor dynamic instability could be produced with a laminar boundary layer on the small airfoil model. It was verified that a turbulent boundary layer on the same model also gave no instability. The results for instability reported herein are therefore entirely for high Reynolds number turbulent flow ahead of the slot. Such tests were carried out by sucking the wall boundary layer from the 16" wall of the C.A.L. 3" x 16" subsonic wind tunnel.

Boundary-Layer Characteristics



For purposes of this analysis, it will be adequate to assume that the boundary-layer velocity profile is given by a power law, namely

$$\frac{u}{u_o} = \left(y/\delta_o\right)^n \qquad --- (1)$$

where δ_o is the boundary-layer "thickness". If the flow entering the slot is smaller than that corresponding to the entire boundary-layer thickness, then q, the volume flow entering the slot per unit time, is given by

$$q = \int_0^\delta u\,dy = \int_0^\delta u_o \left(y/\delta_o\right)^n dy$$

where δ is the thickness of flow within the boundary layer which enters the slot. Thus,

$$q = \frac{u_o \delta_o}{n+1} \left(\delta/\delta_o\right)^{n+1} \qquad --- (2)$$

The mean total head of the air entering the slot is given by

$$H_o = \frac{1}{q}\int_0^\delta u\left(P_o + \tfrac{1}{2}\rho u^2\right) dy$$

where P_o is the static pressure of the flow near the slot or

$$H_o = P_o + \frac{1}{2q}\int_0^\delta \rho u^3 dy \qquad --- (3)$$

Now

$$\int_0^\delta \rho u^3 dy = \rho u_o^3 \int_0^\delta \left(y/\delta_o\right)^{3n} dy = \frac{\rho u_o^3}{3n+1}\,\delta_o \left(\delta/\delta_o\right)^{3n+1}$$

Then

$$H_o = P_o + \frac{\rho u_o^3 \delta_o}{3n+1}\left(\delta/\delta_o\right)^{3n+1} \Big/ 2\,\frac{u_o \delta_o}{n+1}\left(\delta/\delta_o\right)^{n+1}$$

or

$$H_0 = P_0 + \frac{\rho u_0^2}{2}\left(\frac{n+1}{3n+1}\right)\left(\delta/\delta_0\right)^{2n} \qquad \text{--- (4)}$$

From Eq. (2)

$$\delta/\delta_0 = \left[\frac{(n+1)q}{u_0\delta_0}\right]^{\frac{1}{n+1}}$$

Substituting this result into Eq. (4), there results

$$H_0 = P_0 + \frac{\rho u_0^2}{2}\left(\frac{n+1}{3n+1}\right)\left[\frac{(n+1)q}{u_0\delta_0}\right]^{\frac{2n}{n+1}} \qquad \text{--- (5)}$$

From this equation, the value of $\frac{\partial H}{\partial q}$ which is important in considerations of stability may be obtained as

$$\frac{\partial H}{\partial q} = \frac{\rho u_0}{\delta_0}\, n\left(\frac{n+1}{3n+1}\right)\left[\frac{(n+1)q}{u_0\delta_0}\right]^{\frac{n-1}{n+1}} \qquad \text{--- (6)}$$

or in terms of δ/δ_0

$$\frac{\partial H}{\partial q} = \frac{\rho u_0}{\delta_0}\, n\left(\frac{n+1}{3n+1}\right)\left(\delta/\delta_0\right)^{n-1} \qquad \text{--- (7)}$$

If a laminar boundary layer is approximated by $n=1$, the value of $\frac{\partial H}{\partial q}$ is a constant equal to $\frac{\rho u_0}{2\delta_0}$ for all flow rates, as would be expected for a straight line profile. For a turbulent boundary layer (assuming a 1/7 power law), the result is

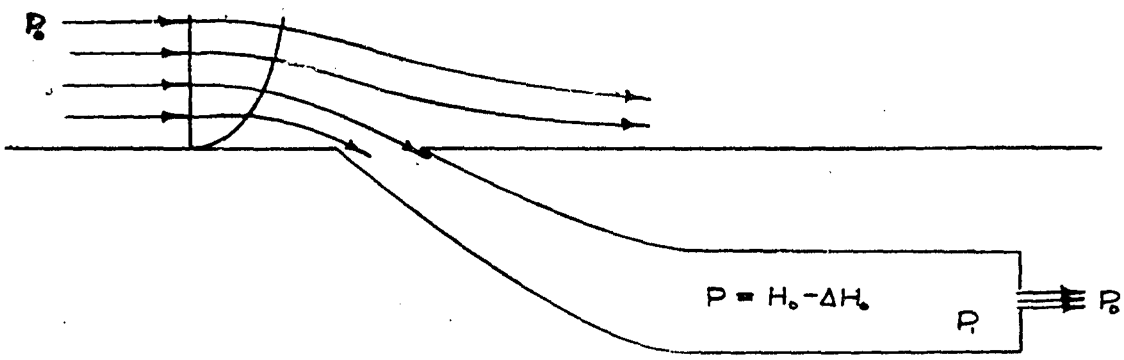
$$\frac{\partial H_0}{\partial q} = \frac{8}{70}\left(\delta/\delta_0\right)^{-\frac{6}{7}}\frac{\rho u_0}{\delta_0} \qquad \text{--- (8)}$$

which slowly decreases with increasing δ or q, and is smaller than the laminar value for the higher flow rates.

As has been indicated in the introduction, instability is associated with positive value of $\frac{\partial H}{\partial q}$. It would, however, be a mistake to conclude from Eqs. (7) and (8) that a flow with laminar profile would be more unstable than one for a turbulent profile, say with $\delta/\delta_0 = 1/2$, because the effects of slot losses have not yet been taken into account. Such losses, which have to do with the mixing and diffusion of the boundary layer air on and after entry into the slot, are not well understood and it appears entirely possible that the losses for a laminar profile could be enough larger than those for a turbulent profile to overcome the larger $\frac{\partial H}{\partial q}$ of the entering air with a laminar profile. Experimental results for slots are inadequate to provide the answer to this question. It has, however, been found experimentally that by proper shaping of the slot and diffuser behind the slot, a considerable portion of the total head of the boundary-layer air could be preserved. In any case, from the above analysis, it may be seen that the range of $\frac{\partial H_o}{\partial q}$ for laminar and turbulent boundary layers is of the same order of magnitude over the middle range of boundary-layer removal $(\delta/\delta_o \approx 1/2)$ and at least on the basis of $\frac{\partial H_o}{\partial q}$, no essential difference in the mechanism of instability should be expected.

Static Stability of the Duct System

By static stability of a flow system is meant the tendency of a system when in a possible steady flow state to return to that state after being slightly disturbed from it by a small (actually infinitesimal) perturbation. The static stability of a system cannot be determined by a consideration of $\frac{\partial H_o}{\partial q}$ for the slot alone, even when losses are included in H_o; the characteristics of the entire system must be considered. As a first example, a very simple system consisting of an entrance slot at a point of pressure P_o, a diffuser, a loss-free duct and an exit orifice also at pressure P_o.



The exit orifice can be assumed to obey the simple pressure drop law (for negligible velocity in the duct)

$$v = \frac{C_e}{\rho}\sqrt{2(P_i - P_o)} \qquad --- (9)$$

where C_e is an orifice coefficient. The flow out the orifice q_e is then given by

$$q_e = \frac{KA_e}{\rho}\sqrt{2(P_i - P_o)} \qquad --- (10)$$

where K is a constant including both the orifice velocity coefficient and the orifice contraction coefficient. The intake characteristics can be given by $H_i = H_o - \Delta H_o$, where H_o is a function of the boundary-layer profile as computed above and ΔH_o is the loss in total head in the slot and diffuser. Roughly, ΔH_o may be expected to be a function of flow quantity squared. The flow in the slot may be represented by q_s. For steady flow, the flow into the system must equal the flow out, and the pressures across the system must balance. Thus

and
$$\left.\begin{aligned} q_e &= q_s \\ H_o - \Delta H_o &= P_i \end{aligned}\right\} \qquad --- (11)$$

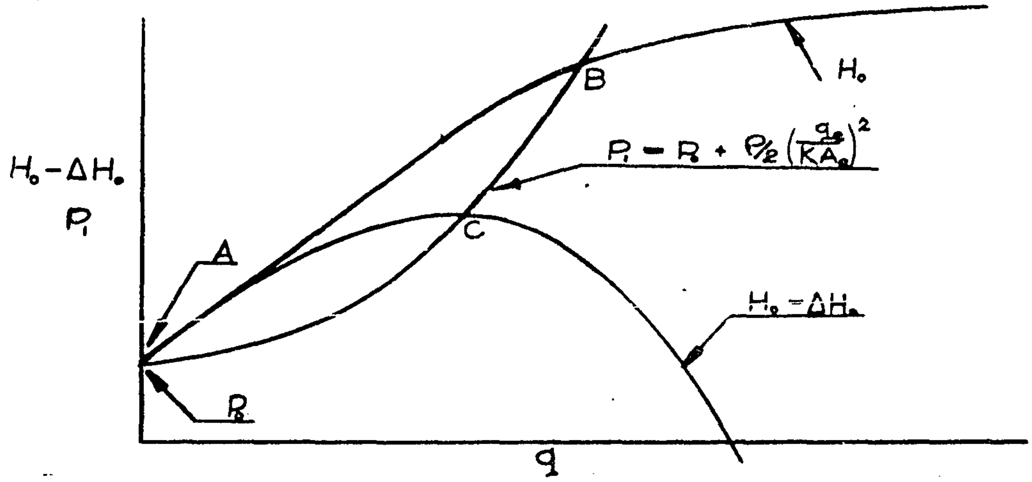
where P_1 is total head at the exit. This may be written in terms of q_e from Eq. (10) as

$$P_1 = P_o + \frac{\rho}{2}\left(\frac{q_e}{KA_e}\right)^2 \qquad --- (12)$$

A steady flow is possible for some value of $q_o = q_e = q$ when

$$H_o(q) - \Delta H_o(q) = P_o + \frac{\rho}{2}\left(\frac{q}{KA_e}\right)^2 \qquad --- (13)$$

Plotting the two sides of this equation versus q leads to a curve like that shown below

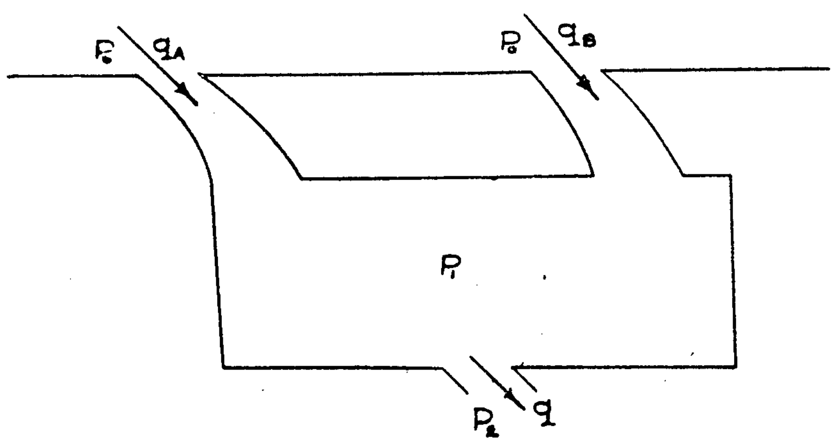


It may be seen that the curve of P_1 intersects the curve of $H_o - \Delta H_o$ at two points, A and C. Point A corresponds to zero flow through the system which is reasonable, since there is no static pressure difference across it while point C corresponds to flow through the system. Point B shows the even larger flow through the system which would be possible if there were no losses at the entry, ΔH_o.

The question now arises of whether the flow which is statically possible at points A and C is also stable at one or both of these points. To answer this question, it is necessary to consider a small increase in flow rate, δq, to occur. This will cause an increase in pressure at the entrance to the duct of amount $\frac{\partial(H_o - \Delta H_o)}{\partial q}\delta q$ and an increase in back-pressure P_1 at the exit of amount $\frac{\partial P_1}{\partial q}\delta q$. Then if $\frac{\partial(H_o - \Delta H_o)}{\partial q}$ is greater than $\frac{\partial P_1}{\partial q}$, the mass of air in the duct will be accelerated toward the exit, thus further increasing the flow through the system. In this case the steady flow is unstable. If on the other hand, $\frac{\partial(H_o - \Delta H_o)}{\partial q}$ is less than $\frac{\partial P_1}{\partial q}$, the flow will be decelerated, since the rise in back-pressure will be greater than the rise in inlet pressure. In this case, the flow is stable, since it tends to return to the original flow rate. From the curves, it may be seen that the slope, $\frac{\partial(H_o - \Delta H_o)}{\partial q}$ is greater than the slope, $\frac{\partial P_1}{\partial q}$, for the condition of no flow at point A, while it is smaller than $\frac{\partial P_1}{\partial q}$ at point C. Thus, the stable condition is that at C, with flow through the system rather than that at A with no flow through the system, in spite of the fact that there is no difference of static pressure across the system. It is important to note that these conclusions as to stability depend upon both the inlet characteristics and the system characteristics, not on the inlet alone; i.e.: for stability, $\frac{\partial(H_o - \Delta H_o)}{\partial q} - \frac{\partial R}{\partial q} < 0$. It is true that for $\frac{\partial(H_o - \Delta H_o)}{\partial q}$ itself less than zero, the system would necessarily be stable at the zero flow condition. On the other hand, even if $\frac{\partial(H_o - \Delta H_o)}{\partial q}$ is positive, the system can be stable in the zero flow condition if $\frac{\partial R}{\partial q}$ is sufficiently large there, which would be the case if the system had large enough losses.

It requires no particular change in the above analysis to imagine that the exit orifice is actually a second slot inlet, since for flow out of the inlet, it would have essentially the flow characteristics of an orifice. Thus, the interconnection of two identical slots by a duct might be expected to lead to a stable condition in which flow enters one slot and leaves through the other, the condition of zero flow through the system being unstable. The particular slot which acts in the inlet would be a matter of random choice, depending on the nature of initial disturbances leading to any given situation.

Actually, the problem of two identical slots discussed above is not of as much practical importance as the one in which there are both two inlet slots and an exit orifice at reduced pressure, which corresponds to the case of boundary-layer suction with two slots, as shown below:



It should be noted here that Norman and Holzhauser (ref. 7) have analyzed a similar system in connection with twin-duct intakes for engines.

For simplicity, $H_o(q)$ and $\Delta H_o(q)$ are assumed to be identical for the two slots A and B. The flow rate through A is q_A, the flow rate through B is q_B, and the total flow is the sum of q_A and q_B. The two diffusers empty into a plenum chamber in which the flow velocity is assumed to be negligibly small. For a steady (but not necessarily stable) flow to be possible,

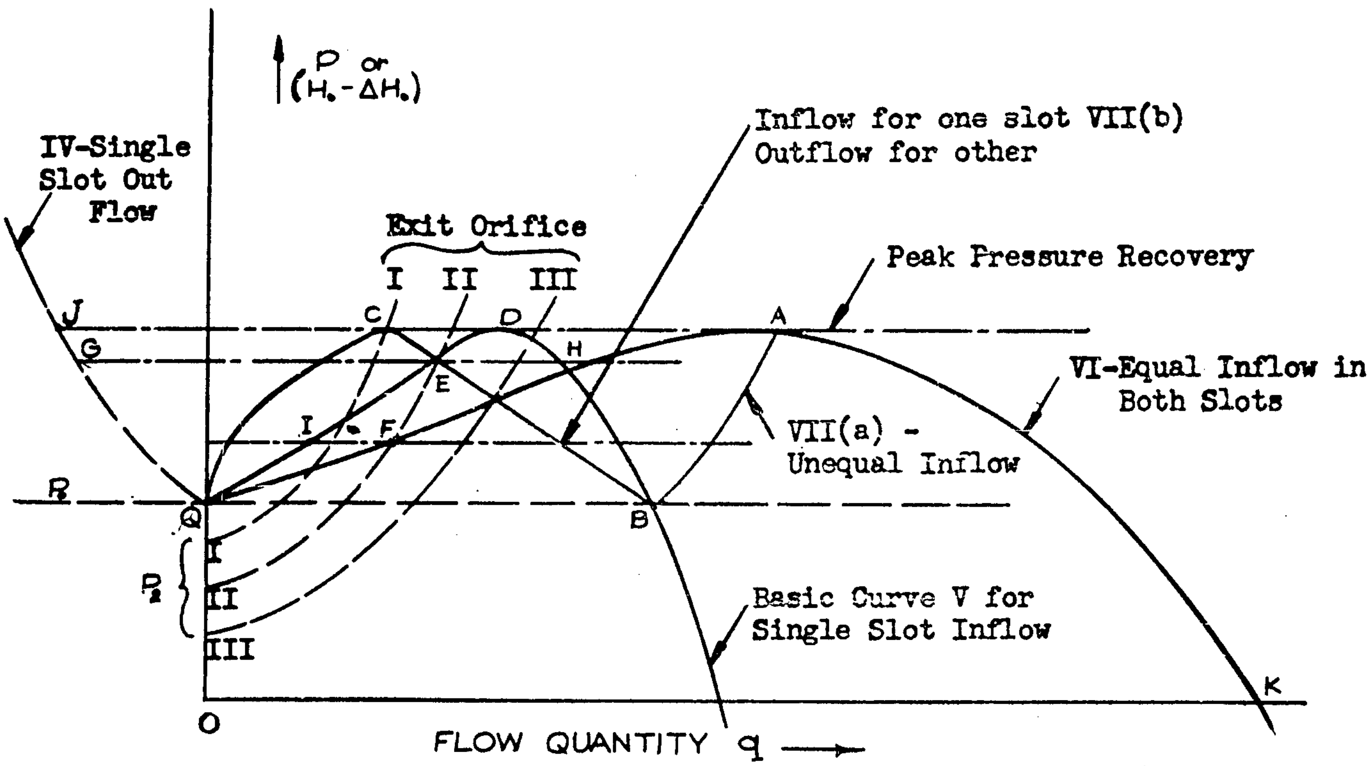
$$q = q_A + q_B$$

$$H_o(q_A) - \Delta H_o(q_A) = P_1$$

$$H_o(q_B) - \Delta H_o(q_B) = P_1$$

$$P_1 = P_2 + \frac{\rho}{2}\left(\frac{q}{KA_e}\right)^2$$

These equations may be represented graphically below. In the curves, due account must be taken of the fact that one or the other of the slots may act as an exit as well as an intake. It may be seen that the situation has become



considerably more complicated than the one previously studied. Nevertheless, the same basic principles apply. As in the simpler case previously treated, the problem can best be discussed in terms of curves of $H_o - \Delta H_o$ versus q and P_1 versus q . The exit orifice curves for three different values of exit pressure, P_2 , Curves I, II and III are simple parabolas corresponding to Eq. (9). Curve IV for outflow through one slot as an exit orifice is similarly plotted as a parabola but for negative q , denoting that it subtracts from the total inlet flow. Curve V is the basic inlet flow curve for a single slot, and has the shape previously noted. Curve VI is the curve for equal flow through the two inlet slots; it is obtained by multiplying the flow quantities for Curve V by 2, Curves VII(a) and VII(b) bounded by points A , B , C and Q are the curves for unequal and inflow, and for inflow in one slot and outflow in the other, respectively. Since the plenum chamber pressure is constant, the values of $H_o - \Delta H_o$, or P_1 (if the flow is out) must be the same for the two slots. Thus, Curve VII(b) is constructed by subtracting from the inlet flow curve for one slot, V, the flow quantities for the single slot outflow curve, IV, at the same pressure.
Curve VII(a) which represents unequal but not reversed flow in the two inlets is constructed by adding the flow rates on the left side of the peak pressure

point D to those on the right side, giving segment A B. Thus, it now appears that for each of the exit orifice curves, there are two solutions for plenum chamber pressure and flow rate, one from the intersection of the exit orifice curves with Curve VI (equal flow) and the other from an intersection with Curve VII (unequal or reversed flow). Of course, if the exit pressure is low enough to produce a total flow rate to the right of point A, there is only one possible flow, not two, corresponding to each inlet operating on Curve V to the right of point D.

It is now necessary to inquire into the question of which solution is stable when two exist. The criterion previously evolved for a single inlet system cannot be directly applied to the curves of total flow versus plenum chamber pressure for the entire system, and such curves, which, incidentally, are those directly obtained from experiment, can give no direct indication of the stability of the system. Instead, it is necessary to consider whether small disturbances in the established flow rates in any branch of the system can upset the established equilibrium. If this be done, it is found analogously to the simple case previously considered that the conditions for stability are

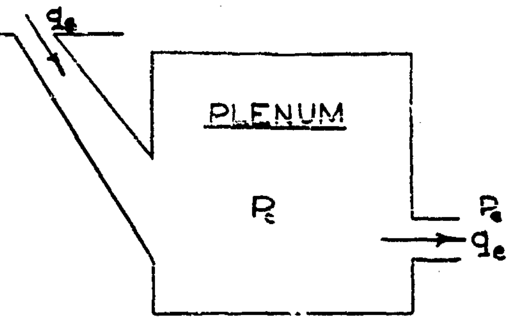
$$\left.\begin{aligned} &\frac{\partial(H_o-\Delta H_o)_A}{\partial q_A} - \frac{\partial R}{\partial q} < 0 \\ &\frac{\partial(H_o-\Delta H_o)_B}{\partial q_B} - \frac{\partial R}{\partial q} < 0 \\ &\frac{\partial(H_o-\Delta H_o)_A}{\partial q_A} + \frac{\partial(H_o-\Delta H_o)_B}{\partial q_B} < 0 \end{aligned}\right\} \quad --- (14)$$

If the flow is reversed in one of the ducts, then the sign of the derivative relating to it in the last equation is reversed. All derivatives are evaluated for the values of the variables corresponding to the equilibrium flow which is being investigated for stability or, in other words, at the values of pressure and flow rate in each branch of the system for the given case.

Consider now the case of exit orifice Curve II which intersects the equal slot flow Curve VI at F and the curve for reversed flow in one slot VII(b) at E. In this case, the point at F which corresponds to both slots operating on the single slot inlet curve at point I is an unstable flow point, since the condition given by Eq. (14) is violated for both slots having positive slopes of the total head versus flow rate curve. On the other hand, point E, which corresponds to one slot operating at point H on the stable part of the single slot curve and the other slot operating on point G of the reversed-flow curve, is stable. Another interesting case is the one exhibited by exit orifice Curve I which shows a stable point C at the peak pressure recovery, which, however, is obtained at the expense of a sizeable outflow from one slot operating at point J on the outflow curve. Thus, in general, the stable operating curve, if one exists, is likely to be of the form of Curve VII (points Q C B A K) and to involve unsymmetrical flow and flow reversal in the ducts up to point A, at which point equal flow in the ducts becomes the only possible flow.

Dynamics of the Duct System

In order to simplify the dynamic analysis of the air-induction system, consideration will be given to motions which are small perturbations from a steady flow. The system to be considered consists of an inlet duct, a plenum chamber of volume, V, and an exit duct, as shown in the figure below. Such a system is closely analogous to the Helmholtz resonator in acoustics. The assumptions made for the analysis of the Helmholz resonator (ref. 8) may be applied here. First, it is assumed that the wave lengths of any oscillations which may occur are large compared to the dimensions of the system. Thus, the assumption of unsteady incompressible flow may be adopted. Second, it is assumed that the velocities in the plenum chamber are negligibly small and that this chamber may be considered to be an air spring. The inertia effects in the system are assumed to be derived from the masses of air in the inlet and exit ducts and from any apparent mass contributions due to the air at the ends of these ducts. The volume flow rates are q_s and q_e for the inlet and outlet ducts respectively. The total pressure at the entrance to the inlet duct is denoted by H_s ; thus, total pressure is assumed to include the effect of all losses in the duct itself as well as the boundary-layer and entrance losses. The pressure immediately ahead of the exit duct is equal to the plenum chamber pressure P_c.



The spring constant of the plenum chamber can now be computed. The mass increment accumulated in the plenum chamber due to incremental flow rate changes q_s and q_e is

$$\delta M_c = \rho \int_0^t (q_s - q_e)\,dt \qquad \text{--- (15)}$$

The corresponding change in density is

$$\delta \rho_c = \frac{\delta M_c}{V} = \rho/V \int_0^t (q_s - q_e)\,dt \qquad \text{--- (16)}$$

where V is the volume of the plenum chamber. The pressure is related to the density by

$$a^2 = \frac{dP}{d\rho} \qquad \text{--- (17)}$$

where a is the velocity of sound. Then $\delta P_c = a^2 \delta \rho_c$, or

$$\delta P_c = a^2 \rho/V \int_0^t (q_s - q_e)\,dt \qquad \text{--- (18)}$$

The term $a^2 \rho/V$ will be called k, so that

$$\delta P_c = k \int_0^t (q_s - q_e)\,dt \qquad \text{--- (19)}$$

The flow in the inlet duct will next be considered as a one-dimensional, unsteady, incompressible flow. The Euler equation is

$$\cancel{\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} + \frac{\partial P}{\partial x} = 0} \qquad \text{--- (20)}$$

Integrating this over the length of the duct, ℓ, gives

$$\int_0^\ell \rho \frac{\partial u}{\partial t}\,dx + \int_0^\ell \rho u \frac{\partial u}{\partial x}\,dx + \int_0^\ell \frac{\partial P}{\partial x}\,dx = 0 \qquad \text{--- (21)}$$

The second integrand is an exact differential, so that the last equation may be written

$$\rho \int_0^{l_s} \frac{\partial u}{\partial t}\, dx + \left(\rho \frac{u^2}{2} + P\right)\Big|_0^{l_s} = 0$$

or

$$\rho \int_0^{l_s} \frac{\partial u}{\partial t}\, dx + P_c - H_s = 0 \qquad --- (22)$$

since $\left(\rho\frac{u^2}{2} + P\right)$ is H_s at the duct entrance and P_c at the duct exit. Since the flow is incompressible, the velocity along the inlet duct is given by

$$u = q_s/A$$

where A is the cross-sectional area of the duct at any point. Thus, the Eulerian equation may be written as

$$\rho \frac{\partial q_s}{\partial t} \int_0^{l_s} \frac{dx}{A} + P_c - H_s = 0 \qquad --- (23)$$

Using A_s as the area of the slot entrance, this may be nondimensionalized to give

$$\frac{\partial q_s}{\partial t}\, \frac{\rho l_s}{A} \int_0^1 \frac{d(x/l_s)}{(A/A_s)} + P_c - H_s = 0 \qquad --- (24)$$

The remaining integral is a dimensionless function of duct geometry only. Using

$$\xi = \int_0^1 \frac{d(x/l_s)}{(A/A_s)} \qquad --- (25)$$

the equation of motion is

$$\left(\frac{\rho l_s \xi}{A_s}\right) \frac{\partial q_s}{\partial t} + P_c - H_s = 0 \qquad --- (26)$$

For the unperturbed motion $P_{c_0} - H_{s_0} = 0$. It is therefore only necessary to consider the changes δP_c and δH_s arising from the perturbation flow. δP_c is given by Eq. (19), while for small perturbations, δH_s is given by

$$\delta H_s = \frac{\partial H_s}{\partial q_s} q_s \qquad --- (27)$$

since, as before q_s represents only the perturbation flow. $\frac{\partial H_s}{\partial q_s}$ is the slope of the characteristic curve for the inlet slot, which has previously been discussed in connection with static instability, $\left[\frac{\partial H_s}{\partial q_s} = \frac{\partial (H_o - \Delta H_o)}{\partial q}\right]$.

Combining Eqs. (19), (26) and (27) leads to the following equation for the inlet duct flow:

$$\left(\frac{\rho l_s \xi_o}{A_s}\right) \frac{\partial q_s}{\partial t} - \frac{\partial H_s}{\partial q_s} q_s + k\int_0^t q_s\, dt - k\int_0^t q_c\, dt = 0 \qquad --- (28)$$

For the outlet duct, a very similar equation can be obtained, namely,

$$\left(\rho\frac{\ell_e \xi_e}{A_e}\right)\frac{\partial q_e}{\partial t} + \frac{\partial B}{\partial q_e} q_e + k\int_0^t q_e\,dt - k\int_0^t q_s\,dt = 0 \qquad --- (29)$$

where ℓ_e, ξ_e, and A_e are now to be computed for the outlet duct and $\frac{\partial B}{\partial q_e}$ is the slope of the characteristic curve of the outlet duct which has been also discussed previously in the section on static instability. Now let

$$\xi_s = \int_0^t q_s\,dt$$

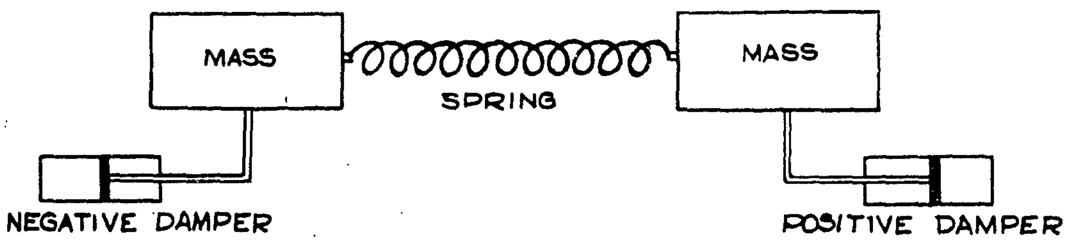
and

$$\xi_e = \int_0^t q_e\,dt \qquad --- (30)$$

Eqs. (28) and (29) then become

$$\left.\begin{aligned}&\left(\rho\frac{\ell_s \xi_s}{A_s}\right)\frac{\partial^2 \xi_s}{\partial t^2} - \left(\frac{\partial H_s}{\partial q_s}\right)\frac{\partial \xi_s}{\partial t} + k\xi_s - k\xi_e = 0\\ &\left(\rho\frac{\ell_e \xi_e}{A_e}\right)\frac{\partial^2 \xi_e}{\partial t^2} + \left(\frac{\partial B}{\partial q_e}\right)\frac{\partial \xi_e}{\partial t} + k\xi_e - k\xi_s = 0\end{aligned}\right\} \qquad --- (31)$$

These may be recognized as analogous to the equations of motion for two masses coupled by a spring and having dampers attached. A positive value of $\frac{\partial H_s}{\partial q_s}$ may be seen to correspond to negative damping of one of the masses while a positive value of $\frac{\partial B}{\partial q_e}$ corresponds to positive damping of the other mass. The system is illustrated below.



<u>Helmholtz Resonator Considerations</u>

If there is no initial flow through the system so that $\frac{\partial H_s}{\partial q_s}$ and $\frac{\partial B}{\partial q_e}$ are effectively zero, Eqs. (31) reduce to the equations for a Helmholtz resonator with two necks (ref. 8). The equations may then be written as

$$\left.\begin{aligned}&\frac{\partial^2 \xi_s}{\partial t^2} + \omega_s^2 \xi_s - \omega_s^2 \xi_e = 0\\ &\frac{\partial^2 \xi_e}{\partial t^2} + \omega_e^2 \xi_e - \omega_e^2 \xi_s = 0\end{aligned}\right\} \qquad --- (32)$$

where

$$\omega_s = \sqrt{\frac{kA_s}{\rho l_s \varsigma_s}} = a\sqrt{\frac{A_s}{V l_s \varsigma_s}}$$

and

$$\omega_e = \sqrt{\frac{kA_e}{\rho l_e \varsigma_e}} = a\sqrt{\frac{A_e}{V l_e \varsigma_e}} \qquad \text{--- (33)}$$

The formulas for ω_s and ω_e are the usual formulas for simple Helmholtz resonators having one neck (ref. 8), except for the factors ς_s and ς_e which are corrections for the shape of the ducts or necks in the resonator analogy. Eqs. (32) may be solved to give the frequency of the resonator with two necks in terms of the frequencies it would have with each neck alone. This frequency ω is given by the solution of

$$\begin{vmatrix} (-\omega^2+\omega_s^2) & -\omega_s^2 \\ -\omega_e^2 & (-\omega^2+\omega_e^2) \end{vmatrix} = 0 \qquad \text{--- (34)}$$

which leads to

$$\omega^4 - \omega^2(\omega_e^2 + \omega_s^2) = 0 \qquad \text{--- (35)}$$

This equation has the roots

$$\omega^2 = 0$$

and

$$\omega^2 = \omega_e^2 + \omega_s^2 \qquad \text{--- (36)}$$

The root $\omega = 0$ corresponds to the fact that air can leak slowly into one neck and out the other, and is of no interest in the present problem. The frequency of interest is the one corresponding to the second root

$$\omega = \sqrt{\omega_s^2 + \omega_e^2} \qquad \text{--- (37)}$$

or

$$\omega = a\sqrt{\frac{1}{V}\left(\frac{A_s}{l_s\varsigma_s} + \frac{A_e}{l_e\varsigma_e}\right)} \qquad \text{--- (38)}$$

If the exit duct length, l_e, is large compared to the inlet duct length, l_s, the term $\frac{A_e}{l_e\varsigma_e}$ is small compared to $\frac{A_s}{l_s\varsigma_s}$, and the frequency is approximately equal to the frequency of a resonator having only the inlet duct as a neck. This was the case in the experiments, which will be discussed later in this report.

For calculations which are made to afford a comparison of theoretical and experimental results, it is necessary to compute the value of ς_s for the duct shape of interest. In the present experiments, the duct was a two-dimensional diffuser with a linear expansion of area from the slot to the plenum chamber. Taking the ratio of areas at the two ends to be n, the area at any section a distance x from the slot is given by

$$A = A_s\left[1 + (n-1)\frac{x}{l_s}\right] \qquad \text{--- (39)}$$

The integral for ξ_s is then given by

$$\xi_s = \int_0^1 \frac{d(x/l_s)}{1+(n-1)\,x/l_s} = \frac{1}{(n-1)} \ln\left[1+(n-1)\,x/l_s\right]\Big|_0^1 = \frac{\ln(n)}{n-1} \qquad \text{--- (40)}$$

Substituting this into Eq. (33) gives for the frequency

$$\omega_s = a\sqrt{\frac{A_s\,(n-1)}{V l_s \ln(n)}} \qquad \text{--- (41)}$$

Dynamic Stability Conditions

The linearized differential equations of motion for small perturbations of the duct system, Eqs. (31) may be analyzed to determine the criteria for dynamic stability of the system. Since the stability is defined in terms of response to infinitesimal disturbances, these stability criteria based on the small-perturbation equations are exact to the extent that the physical parameters involved in the coefficients of the differential equation can be determined exactly. The solutions to the differential equations which describe the motion of the system are, on the other hand, not exact for non-linear systems, such as those being considered here. The accuracy of the calculated frequency, for instance, depends on the amplitude of any motions described being small enough.

The equations of motion, Eqs. (31), may be written as

$$\left.\begin{aligned} \frac{d^2\xi_s}{dt^2} + b_s\frac{d\xi_s}{dt} + \omega_s^2\,\xi_s - \omega_s^2\,\xi_e &= 0 \\ \frac{d^2\xi_e}{dt^2} + b_e\frac{d\xi_e}{dt} + \omega_e^2\,\xi_e - \omega_e^2\,\xi_s &= 0 \end{aligned}\right\} \qquad \text{--- (42)}$$

where

$$\left.\begin{aligned} b_s &= -\frac{\partial H_s}{\partial q_s}\left(\frac{A_s}{\rho l_s \xi_s}\right) \\ b_e &= \frac{\partial R}{\partial q_e}\left(\frac{A_e}{\rho l_e \xi_e}\right) \end{aligned}\right\} \qquad \text{--- (43)}$$

The solutions are of the form

$$\left.\begin{aligned} \xi_s &= c_1 e^{pt} \\ \xi_e &= c_2 e^{pt} \end{aligned}\right\} \qquad \text{--- (44)}$$

and the characteristic equation is then

$$(p^2 + b_s p + \omega_s^2)(p^2 + b_e p + \omega_e^2) - \omega_e^2\omega_s^2 = 0 \qquad \text{--- (45)}$$

or

$$p^4 + (b_s + b_e)p^3 + (\omega_s^2 + \omega_e^2 + b_s b_e)p^2 + (\omega_s^2 b_e + \omega_e^2 b_s)p = 0$$

Factoring out the root $P=0$ leads to the cubic

$$P^3 + (b_s + b_e)P^2 + (\omega_s^2 + \omega_e^2 + b_s b_e)P + (\omega_s^2 b_e + \omega_e^2 b_s) = 0 \qquad --- (46)$$

According to Routh's criteria (ref. 9), P will not have a positive real part (i.e., the system will be stable) if all the coefficients of the equation are positive and if, for the equation

$$P^3 + BP^2 + CP + D = 0, \qquad BC - D > 0 \text{ (Routh's Discriminant)} \qquad --- (47)$$

or in this case

$$(b_s + b_e)(\omega_s^2 + \omega_e^2 + b_s b_e) - (\omega_s^2 b_e + \omega_e^2 b_s) > 0$$

This can be simplified to

$$\omega_s^2 b_s + \omega_e^2 b_e + b_s b_e (b_s + b_e) > 0 \qquad --- (48)$$

The requirement that the coefficients of the characteristic equation all be positive is sufficient to insure static stability of the type discussed previously. In particular, the requirement that the constant coefficient of the equation must be positive leads to

$$\omega_s^2 b_e + \omega_e^2 b_s > 0$$

or

$$\left[-k \frac{\partial H_s}{\partial q_s} + k \frac{\partial P}{\partial q_e}\right]\left(\frac{A_s}{\rho \ell_s \xi_s}\right)\left(\frac{A_e}{\rho \ell_e \xi_e}\right) > 0 \qquad --- (49)$$

This finally is reduced to

$$-\frac{\partial H_s}{\partial q_s} + \frac{\partial P}{\partial q_e} > 0 \qquad --- (50)$$

which, with due account of changes in notation, is equivalent to Eqs. (14) for static stability in the same case. Since a real cubic has either three real roots or a real root and two complex conjugate roots, and since the constant coefficient is equal to minus the product of the roots, a negative value of this coefficient indicates the existence of at least one real positive root. This represents a non-oscillating divergent type of instability which is what would be expected in the case of static instability.

If the coefficient of P^2 is negative but the constant coefficient is not, (i.e., the system is statically stable) the instability is oscillatory divergent. This is clear from the fact that the coefficient of P^2 is equal to minus the sum of the roots of the cubic, so that if there is no positive real root, the coefficient can only be negative if the real part of the pair of complex conjugate roots is positive. The requirement is

$$b_s + b_e > 0$$

or

$$-\frac{\partial H_s}{\partial q_s}\left(\frac{A_s}{\rho \ell_s \xi_s}\right) + \frac{\partial P}{\partial q_e}\left(\frac{A_e}{\rho \ell_e \xi_e}\right) > 0 \qquad --- (51)$$

This is similar to the condition for static stability except that the damping term for each duct is divided by the mass term for that duct. This is connected with the fact that in the analogous oscillatory motion of two masses coupled by a spring, the amplitudes of the two elements are inversely proportional to their masses. Thus, a damper on the larger mass is less effective than one on the smaller mass. Similarly, in the present case, high exit losses ($\frac{\partial P}{\partial q_e}$ large), which may be sufficient to provide static stability according to Eq. (50) may be ineffective in providing dynamic stability according to Eq. (51), if the effective exit mass, $\left(\frac{A_e}{\rho l_e \xi_e}\right)$, is large compared to the effective inlet mass.

Finally, if all the coefficients of the characteristic equation are made positive, the sole criterion for stability is Routh's discriminant, Eq. (48). Substituting the values of ω_s, ω_e, b_s and b_e into the first two terms of this equation, there results

$$k\left[-\frac{\partial H_s}{\partial q_s}\left(\frac{A_s}{\rho l_s \xi_s}\right)^2 + \frac{\partial P}{\partial q_e}\left(\frac{A_e}{\rho l_e \xi_e}\right)^2\right] + b_s b_e (b_s + b_e) > 0 \qquad --- (52)$$

Thus, if $\frac{\partial H_s}{\partial q_s}$ is positive (b_s negative) while $-\frac{\partial H_s}{\partial q_s} + \frac{\partial P}{\partial q_e} > 0$ (static stability condition) and $(b_s + b_e)$ is also positive, this condition can only be satisfied if the first term is sufficiently positive. In order for this term to be positive at all, the sum of the damping coefficients divided by the squares of the mass coefficients must be positive. This is an even more severe requirement for large $\frac{\partial P}{\partial q_e}$ (large exit loss) than Eq. (51), if the effective mass of the exit duct is relatively large. If the term in brackets is positive, then increasing k (which amounts to decreasing the plenum volume) is favorable for dynamic stability.

Non-Linear Effects

From the stability criteria given above, it may be seen that if the effective mass of the exit duct is large, a statically stable system can easily be dynamically unstable. According to the linearized equations given above, the amplitude of oscillation would increase in time without limit. Actually, due to the nonlinearity of the system, the oscillations will settle down to a periodic motion at some finite amplitude. This motion is called a "limit cycle" in nonlinear mechanics. This is, of course, what is actually observed in practical cases of dynamic instability. To investigate qualitatively the nature of the limit cycle, it is convenient to consider a case in which the effective mass of the exit duct is so high that it experiences relatively little amplitude of oscillation compared to the inlet duct. In this case, the motion may be considered as that of the inlet duct only. The equation of motion is then

$$\ddot{\xi}_s + b_s(\dot{\xi}_s, q_o)\dot{\xi}_s + \omega_s^2 \xi_s = 0 \qquad --- (53)$$

where $b_s(\dot{\xi}_s, q_o)$ is a function of the mean flow rate q_o and the instantaneous value of $\dot{\xi}_s$, and ω_s^2 is considered to be constant. If this equation be multiplied by $\dot{\xi}_s$ and integrated over a period of the motion, T, there results

$$\int_0^T \ddot{\xi}_s \dot{\xi}_s \, dt + \int_0^T b_s(\dot{\xi}_s, q_o)(\dot{\xi}_s)^2 dt + \omega_s^2 \int_0^T \xi_s \dot{\xi}_s \, dt = 0 \qquad --- (54)$$

This is equivalent to

$$\int_0^T \dot{\xi}_s \, d\dot{\xi}_s + \omega_s^2 \int_0^T \xi_s \, d\xi_s + \int_0^T b_s(\dot{\xi}_s, q_o)(\dot{\xi}_s)^2 dt = 0$$

or

$$\left[\tfrac{1}{2}(\dot{\xi}_s)^2 + \omega_s^2 \, \xi_s^2/2 \right]_0^T + \int_0^T b_s(\dot{\xi}_s, q_o)(\dot{\xi}_s)^2 dt = 0$$

The first term is zero if T is the period, so the condition for periodic motion is

$$\int_0^T b_s(\dot{\xi}_s, q_o)(\dot{\xi}_s)^2 dt = 0 \qquad \text{--- (55)}$$

This is what might have been expected; it states that the average energy input to the system over a cycle must be zero. The condition may be written as

$$\int_0^T b_s(\dot{\xi}_s, q_o)\, \dot{\xi}_s \, d\dot{\xi}_s = 0 \qquad \text{--- (56)}$$

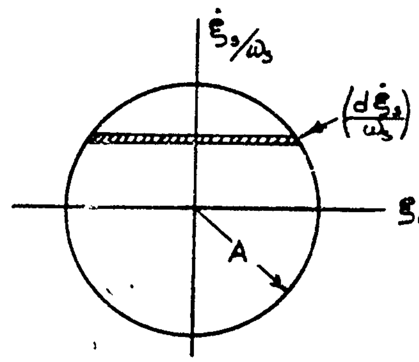
If b_s is small compared to ω_s, the solution of Eq. (53) will be very nearly sinusoidal, namely,

$$\xi_s = A \sin \omega_s t$$

and

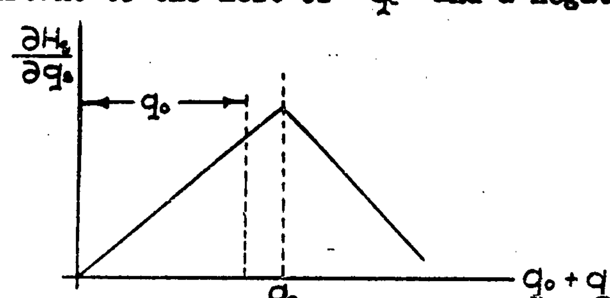
$$\dot{\xi}_s = A\omega_s \cos \omega_s t$$

The curve of $\dot{\xi}_s/\omega_s$ versus ξ_s will then be a circle of radius A as shown in the figure below.



The integral will be equal to the limit of the sum of each elementary strip $\frac{d\dot{\xi}_s}{\omega_s}$ multiplied by the value of $\omega_s b_s$ appropriate to that value of ξ_s.

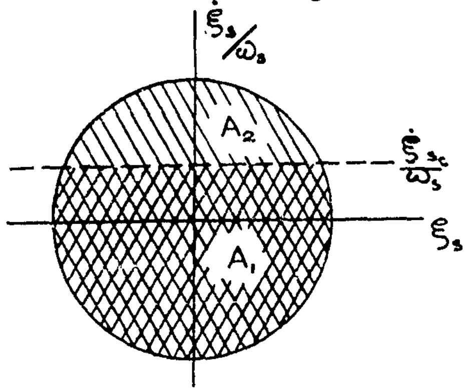
As a very simple case, consider a curve of $\frac{\partial H_s}{\partial q_s}$ of the type shown below. The slope is a positive constant to the left of q_c and a negative constant to the right.



Here q_o is the mean flow rate and q_c is the point on the curve where $\frac{\partial H_s}{\partial q_s}$ changes from positive to negative. Then at $\dot{\xi}_{s_c} = q_c - q_o$, b_s changes from positive to negative. Thus, on a diagram of $\dot{\xi}_s/\omega_s$ versus ξ_s , the limit cycle corresponds to the case when

$$b_{s_1}A_1 + b_{s_2}A_2 = 0 \qquad \text{--- (57)}$$

where b_{s_1} is the value of b_s for $\dot{\xi}_s < \dot{\xi}_{sc}$ and b_{s_2} is the value for $\dot{\xi}_s > \dot{\xi}_{sc}$ while A_1 and A_2 are the areas shown on the diagram below.



It is obvious from these considerations that in some cases oscillation is possible even when q_o is large enough so that $\frac{\partial H_s}{\partial q_s}$ is negative. In this case, the flow would be dynamically stable for small disturbances, but a large disturbance of the right kind could start an oscillation of finite amplitude.

DESCRIPTION OF MODELS AND EXPERIMENTAL APPARATUS

The experimental work was carried out in the Cornell Aeronautical Laboratory 3" by 16" subsonic induction tunnel, which is of the non-return type. This tunnel is powered by the 600 horsepower Nash pumps used for pressurization of the C.A.L. $8\frac{1}{2}$' by 12' Variable Density Tunnel which are used to supply the ejector jets for this tunnel. Most of the tests were made using the turbulent boundary layer on the 16" wall of this tunnel as the induction air. A slot 13.75" wide was made in the wall of the tunnel and opened into a diffuser section leading to the plenum chamber. The boundary-layer air was evacuated from the plenum chamber through a line connected to a 40 horsepower Kinney positive displacement vacuum pump. A bypass valve was also provided in this line to allow suction of air from the room along with that from the plenum chamber. This bypass permitted a wider range of suction pressures and volume flows than was possible with a direct connection to the pump. It also permitted dynamic conditions at the exit orifice which were not as strongly dependent on the vacuum pump characteristics as with a closed line. The general arrangement of the wind tunnel, plenum chamber, and vacuum line is shown in Figs. 1 and 2.

The slot entry and diffuser were designed to provide good pressure recovery of the entering boundary-layer air. Based on tests by Pierpont (ref. 10), the slot shape and diffuser design shown in Fig. 3 were selected. The area ratio of the diffuser between plenum chamber entrance and throat was, however, much larger than that used in Pierpont's tests in order to avoid the loss of dynamic head which would otherwise occur at the plenum entrance. The diffuser was two-dimensional and was formed by two blocks. The forward block was fixed, while the aft block was moveable to provide for variable slot width. The diffuser area ratio varied from 16 at a slot width of 3/16" to 7.5 at a slot width of 7/16". By contrast, Pierpont's area ratio was 2.0. The diffuser half-angle remained fixed at 6°, since the displacement of the aft block involved no rotation of the wall.

Total head surveys were made to obtain the boundary-layer velocity profiles 2.75" ahead of the slot along the wall centerline. Also, total-head tubes and reverse-total-head tubes were located in the diffuser at four stations along the slot span to check flow uniformity in the slot. This is shown in Fig. 4. Tunnel static pressures were measured by means of wall static taps, and plenum chamber pressure was measured at several points in the plenum chamber. These pressure readings were indicated on a manometer board and recorded photographically. The indications were relatively steady in the presence of high-frequency oscillations because of the damping effect of the long pressure lines and the manometer itself. Mass flow from the plenum chamber was measured by means of a calibrated Venturi tube.

Oscillating pressures were measured by means of a Statham Type P-6 dynamic pressure pickup of the strain gage type and recorded on a Brush oscillograph. At various times, the dynamic pickup was vented to various pressure taps in the system in order to obtain a sampling of dynamic data at all points. Near the end of the program the Brush oscillograph was unavailable and some data were taken with a carbon microphone, amplifier and cathode-ray oscilloscope. This arrangement turned out to be less satisfactory because of its sensitivity to noise in the tunnel and compressor system. All dynamic data presented in this report is based on measurements with the pressure pickup.

In order to experiment with laminar boundary-layers, a second model was constructed. This was of airfoil shape with a flat lower surface. The suction slot and diffuser were scaled-down versions of those used in the wall model, and were located 12 inches from the airfoil leading edge. The model spanned the tunnel along the 16-inch dimension and had its lower surface 3/4 inch from one of the walls. The aft portion of the model was used as the plenum chamber. The plenum chamber was connected to the gate valve, Venturi tube and vacuum pump through a pair of 3/4 inch I.D. rubber tubes connected at each side of the airfoil. A schematic diagram of the test set-up is shown in Fig. 5.

EXPERIMENTAL RESULTS AND DISCUSSION

Tunnel Wall Boundary Layer-Wide Span Slots

The first tests conducted in this program were made with 13.75 inch span slots on the tunnel wall. Typical boundary-layer velocity profiles ahead of the slot are shown in Fig. 7. Both static and dynamic instability were encountered in these tests for flow rates lower than that required to suck the entire boundary layer across the slot span. The static instability took the form of a nonuniform distribution of flow across the diffuser. At low flow rates, the flow actually reversed at one end of the slot. A typical set of results is shown in Fig. 8. In this case, the boundary layer thickness was about .68 inches. The results shown are for suction flow rates corresponding to sucking .04 inches and .2 inches of the boundary layer. In a crude way, the static stability analysis given earlier for a two-slot system may be applied to a system with a single wide slot by assuming it to be divided in two. It is then clear that the more stable flow at low flow rates would, in fact, be one in which the flow rates in the two sides of the system is unequal. The asymmetry of the flow along the span decreased as the suction flow quantity increased. This effect is better illustrated for a typical case in Fig. 9 in terms of the asymmetry index J defined as

$$J \equiv \frac{u_{max} - u_{min}}{Q/A}$$

where u_{max} and u_{min} are the maximum and minimum velocities measured across the diffuser, Q is the total volume flow rate through the system, and A is the area of the diffuser at the section where the measurements were made. It may be seen that for flow rates approaching half those necessary to suck the entire boundary layer, the asymmetry becomes small. According to the theory for two-slot systems previously presented, this should occur when the peak of the plenum pressure recovery curve has been passed for the flow in both parts of the slot. It proved to be difficult to verify this theoretical requirement because the details of the three-dimensional flow in the slot do not fit the simplifying assumptions of the two-slot theory, and because whenever this type of static instability occured in the present tests, it was also accompanied by dynamic instability with attendant oscillations. It was, however, observed that the diminution of asymmetries occurredat flow rates where the slope of the curve of plenum chamber pressure versus flow rate was negative.

As mentioned above, the wide-span slots all showed both static instability and dynamic instability simultaneously. A typical plenum chamber pressure record made with a dynamic pressure pickup is shown in Fig. 11; it may be seen that, although the record is fairly regular and periodic, it is not sinusoidal. This is evidence of the nonlinearities in the system and was to have been expected. In spite of this fact, the Helmholtz resonator frequencies calculated on the basis of the linearized theory (Eq. 41) agreed fairly well with measured frequencies in most cases. A comparison of calculated and measured frequencies is given in Table I.

TABLE I

COMPARISON OF CALCULATED AND MEASURED OSCILLATION FREQUENCIES

SLOT SPAN	SLOT WIDTH	CALCULATED FREQUENCY (cps)	MEASURED FREQUENCY (cps)
13.75 ins.	3/16 in.	30	27.0 ±.5
13.75 ins.	5/16 in.	33	28.5 ±.5
13.75 ins.	7/16 in.	37	30.0 ±.7
7.85 ins.	5/16 in.	25	21.5 ±.5
3.85 ins.	5/16 in.	18.5	13 -20

The calculated frequencies according to Eq. (41) are based on the assumption that the effective mass of the exit duct is very large compared to the effective mass of the inlet duct. If this were true, the frequency of the system with a 7.85 in. span slot should be $\sqrt{\frac{13.75}{7.85}}$ (= 1.32) times the frequency of the same system with a 13.75 in. span slot. The ratio of the measured frequencies from Table I is $\frac{28.5}{21.5} = 1.32$, which provides a good check of this assumption. The uncertainty of the measured frequency is associated with a generally small rise in frequency with flow rate. This effect became most pronounced for the 3.85 in. span slot listed in Table I for which the frequency varied from 13 to 20 cps. This effect is another indication of the nonlinearity of the system. In order to check on the possibility that the flow in the wind tunnel might be coupled to the plenum chamber oscillation, tests were run with varying lengths of tunnel, but the effects were generally small.

The ranges of dynamic stability observed in the tests of the wide-span slots are shown in Fig. 10 on the curves of plenum chamber pressure versus flow rate. Test points on these curves are shaded if oscillation occurred. Also shown on this figure is a curve of the boundary-layer profile and a curve of flow rate through the system versus amount of boundary layer which would be removed if the flow were uniform,in terms of distance from the wall of the outermost streamline entering the slot. It may be observed that for all slot widths, the plenum pressure drops with flow rate for small flow rates and the system is at first stable. The reasons for this effect have not been determined. It may be that for small flow rates with large asymmetry along the slot,mixing losses in the diffuser and separation of the low-energy air from the diffuser wall are responsible. On the other hand, it may be that this effect is simply a dynamical one, due to the fact that with asymmetrical flow such as might be encountered in this flow regime, the losses associated with the reversed flow are sufficient to stabilize the system. The data at hand are insufficient to settle this question. In any case, for all slot widths, the oscillations do not start until the slope of the curve of plenum chamber pressure versus mass flow has become positive. The oscillations do not necessarily stop, however, when the slope has become negative. This should not be surprising, since as on the segment BC of the curve for the two-slot model, discussed on page 8 , the plenum chamber total pressure may show a falling pressure characteristic while one or the other of the slots is operating on a rising pressure characteristic. If any element of the system has such a positive slope of the total head versus mass flow, then instability is possible. Since a wide-span single slot is potentially a multiple-slot system if spanwise variations of flow rate occur, the two-slot model may provide some insight into its operation. In fact, ignoring the initial pressure drop in the curves, the curves of Fig. 10 show a certain vague similarity to the theoretical two-slot system curve on page 8 , in that the plenum pressures show two maximum points. The differences between the two-slot model and a wide-span slot are not negligible, since mixing between the air entering and leaving the system can take place at high velocity in the diffuser instead of at negligible velocity in the plenum chamber. Thus, the plenum chamber pressure should be lower than would be expected on the basis of the total head of the entering air and estimated slot losses. Further, even if the slot flow were truly two-dimensional, it would not be known how the losses in unsteady flow are related to the steady-flow losses.

In Fig. 12 are presented curves of plenum pressure versus flow rate for a 3/16 in. wide wide-span slot with a throttling device in the diffuser (as shown in Fig. 5). Here again, points at which instability was encountered are shaded. It may be seen that ranges of dynamic instability occur whenever the curve of plenum chamber pressure versus flow rate has a positive slope. With the throttling valve full closed (which still allowed passage of some flow), this slope was always negative and no instability occurred.

Tunnel Wall Boundary Layer - Slots with Splitters

In an attempt to improve the flow uniformity and stability of the wide-span slots, the slot entry and diffuser were divided into four equal sections, each of 3.85 in. span, by means of splitter plates. The plenum was not divided, however. Tests of this arrangement showed no dynamic instability, but there was static instability, evidenced by nonuniform flow rates through the various sections of the duct. In fact, for very small flow rates, all of the flow tended to go through one section of the duct with no flow or a small reverse flow through the other sections. Various sections of the duct would be operative for one total flow rate and inoperative for other flow rates in a random fashion. Once the flow rate was equal to the flow necessary to suck the entire boundary layer across the span of the duct, all sections became operative and uniform flow was approached. Typical results for measured diffuser velocities in the four sections of the duct are given in Table II.

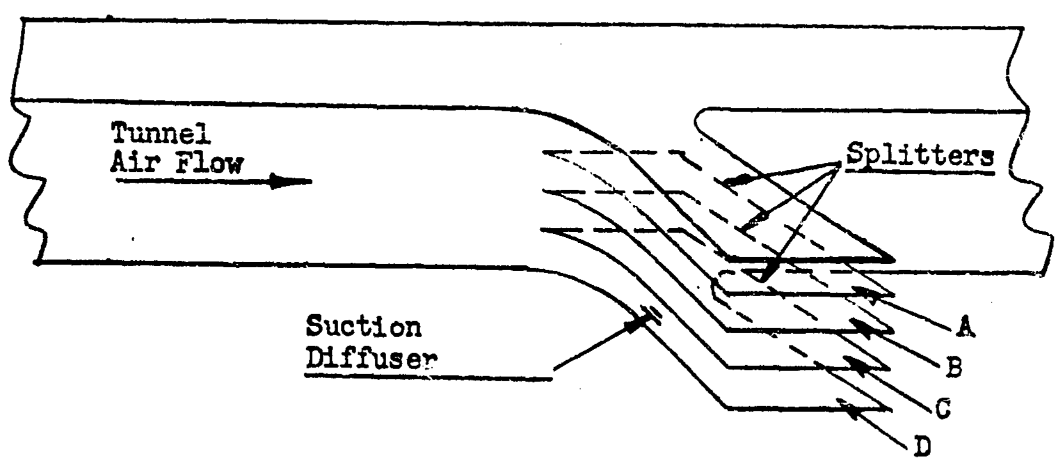


TABLE II

DIFFUSER VELOCITIES FOR VARIOUS FLOW RATES - FT./SEC.

Tunnel $V \approx 270$ ft/sec $Q \sim ft^3/sec$

Q	0.857				2.00				3.78				6.05			
A	154	144	163	159	155	204	201	206	180	210	202	161	185	170	155	126
B	0	0	0	0	0	0	0	0	112	150	146	158	150	149	178	166
C	0	0	0	0	151	162	167	161	0	0	0	0	165	145	180	160
D	0	0	0	0	0	0	0	0	141	175	189	206	118	175	0	196

It may be noted that a quantity of flow in an active section tended to be approximately equal to the flow which would just take in all of the boundary-layer air ahead of the section (one-fourth of the total boundary layer air ahead of the entire slot). Thus, at Q = .857 (ft^3/sec), one section was active; at Q = 2.00, two sections were active; at Q = 3.78, three sections were active; and at Q = 6.05, which is just about enough to remove the boundary-layer air ahead of the slot, four sections were active.

It was desired to verify that the elimination of the dynamic instability was not simply due to the increased losses arising from skin friction on the splitter plates, although this seemed unlikely. This could easily be done by sealing three of the four sections of the duct and applying suction to the fourth. It was found that, with three sections sealed, dynamic instability reappeared with oscillations at a frequency appropriate to the reduced slot area. Curves of plenum chamber pressure versus flow rate are presented in Fig. 13 for a 5/16 in. slot with the splitters, without splitters, and with splitters and three sections sealed. The curves clearly show that effect of the splitters on dynamic stability is not an effect due to increasing duct losses, but is a multiple-slot effect. Apparently what happens is that the flow is statically stable only under assymetrical flow conditions which call for almost complete boundary-layer suction through operative sections of the duct. Under these conditions, the flow is dynamically stable.

Airfoil Model Boundary Layer

In an effort to obtain experimental results on the stability of a duct system ingesting a laminar boundary layer, tests were run with the airfoil model described above (Fig. 6) at tunnel speeds between 40 and 100 ft/sec both with and without stilling screens in the tunnel. A slot width of .05 in. was employed corresponding to the anticipated boundary-layer thickness at 100 ft/sec. It was found that neither static nor dynamic instability could be observed. Tests were then also made at 100 ft/sec with a transition wire on the airfoil to produce a turbulent boundary layer and again no instability was observed. It appears that this stability, as compared with the larger model, may have been due to (1) high slot and duct losses at the small scale of these tests, (2) relatively high exit losses as compared to the larger model or (3) relatively low plenum chamber volume compared to the larger model. Within the limited scope of the present program, it was not possible to embark on the comprehensive tests necessary to determine the effects of all these variables. Items (2) and (3) could be checked within a limited range by using a partial-span slot to reduce the inflow to the system. The slot span was reduced to one-fourth of its original 14 in. value, but the system remained stable.

CONCLUSIONS

Theoretical and experimental studies of elementary air-induction systems for boundary-layer suction lead to the following conclusions:

(1) Systems which include inlets having a rising slope of the curve total pressure recovery with increasing volume flow through the inlet may experience both static and dynamic instability.. High-pressure-recovery inlets ingesting boundary-layer air usually will have this characteristic at low flow rates.

(2) Static instability is usually observed as in the form of unequal flow rates in ostensibly identical branches of the system. Dynamic instability occurs in the form of regular periodic oscillation of the system.

(3) Both static and dynamic instability are usually eliminated once all inlets are operating at flow rates well in excess of that for maximum pressure recovery. It appears, however, that instability may persist somewhat past the point of peak recovery pressure due to nonlinearity of the system and the presence of finite disturbance.

(4) Both static and dynamic instability can usually be eliminated by the introduction of throttling devices producing a pressure drop of the order of the dynamic head of the entering air.

(5) Although the static and dynamic instability have a common basic cause, namely the inlet characteristic, the necessary and sufficient conditions for their occurrence are quite different, and depending on the dynamic characteristics of the system, either, both, or neither may be encountered when the inlet characteristic is unstable.

(6) Static instability of the flow into wide-span slots occurred for typical inlets tested in this program. At very low flow rates, the flow was reversed at one end of the slot and entering at much higher than average speed. At the same time, the flow was oscillating.

(7) The introduction of splitter plates in the wide-span slot and diffuser eliminated the dynamic instability but intensified the static instability at low flow rates, in that the flow tended to enter only some of the sections into which the duct was split. When the total flow entering the slot approached to total boundary-layer quantity ahead of the slot, the flow in the various sections was uniform.

(8) When regular oscillation of the duct system occurred, it was at a frequency generally in good agreement with the calculated Helmholtz resonator frequency of the system.

BIBLIOGRAPHY

1. Pfenninger, W. Experiments on a Laminar Suction Airfoil of 17 Per Cent Thickness. Journal of the Aeronautical Sciences. Volume 16, Number 4. April 1949. pp. 227-236.

2. Dickinson, H.B. Flight and Tunnel Test Research on Boundary-Layer Control. Journal of the Aeronautical Sciences. Volume 16, Number 4. April 1949. pp. 243-251.

3. Smith, A.M.O. and Roberts, Howard E. The Jet Airplane Utilizing Boundary-Layer Air for Propulsion. Journal of the Aeronautical Sciences. Volume 14, Number 2. February 1947. pp. 97-109.

4. von Doenhoff, A.E. and Loftin, L.K. Jr. Present Status of Research on Boundary-Layer Control. Journal of the Aeronautical Sciences. Volume 16, Number 12. December 1949. pp. 729-740.

5. Den Hartog, J. P. Mechanical Vibrations. Second Edition. McGraw-Hill Book Company, Inc. 1940.

6. Bullock, Robert O., Wilcox, Ward W., and Moses, Jason J. Experimental and Theoretical Studies of Surging in Continuous-Flow Compressors. NACA TR 861. 1946.

7. Martin, Norman J. and Holzhauser, Curt A. Analysis of Factors Influencing the Stability Characteristics of Symmetrical Twin-Intake Air-Induction Systems. NACA TN 2049. March 1950.

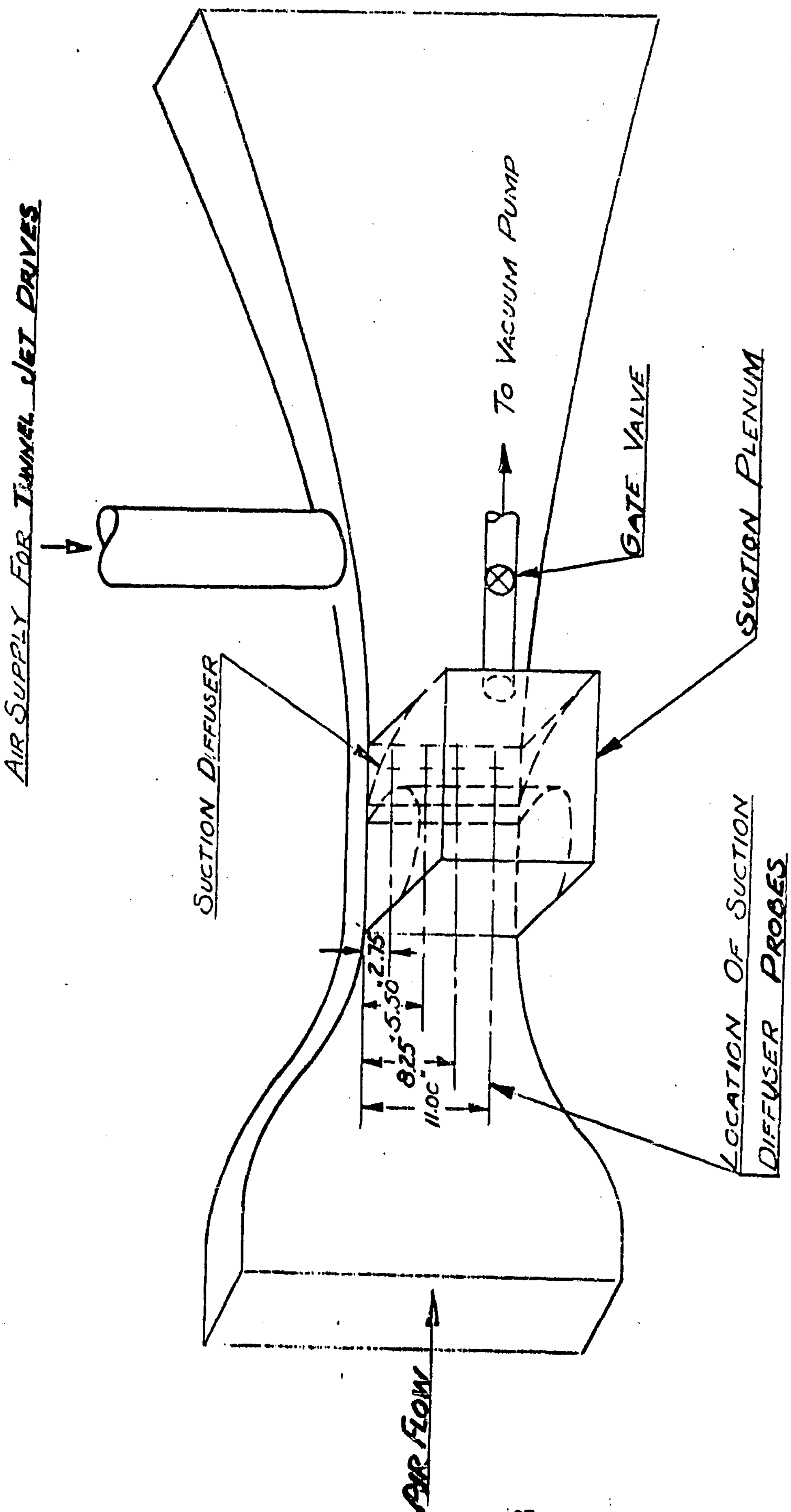
8. Rayleigh, John William Strutt, Baron. The Theory of Sound. Second Edition. Revised and Enlarged. Dover Publications. 1945.

9. von Kármán, Theodor and Biot, Maurice A. Mathematical Methods in Engineering; An Introduction to the Mathematical Treatment of Engineering Problems. First Edition. McGraw-Hill Book Company, Inc. 1940.

10. Pierpont, P.K. Investigation of Suction-Slot Shapes for Controlling a Turbulent Boundary Layer. NACA TN 1292. June 1947.

11. Letter from A.M.O. Smith, Douglas Aircraft Company, Inc. to J. M. Gwinn, Cornell Aeronautical Laboratory, Inc. dated December 1949.

FIG. 1



GENERAL ARRANGEMENT OF WIND TUNNEL AND PLENUM CHAMBER FOR WALL BOUNDARY-LAYER SUCTION

FIG. 2

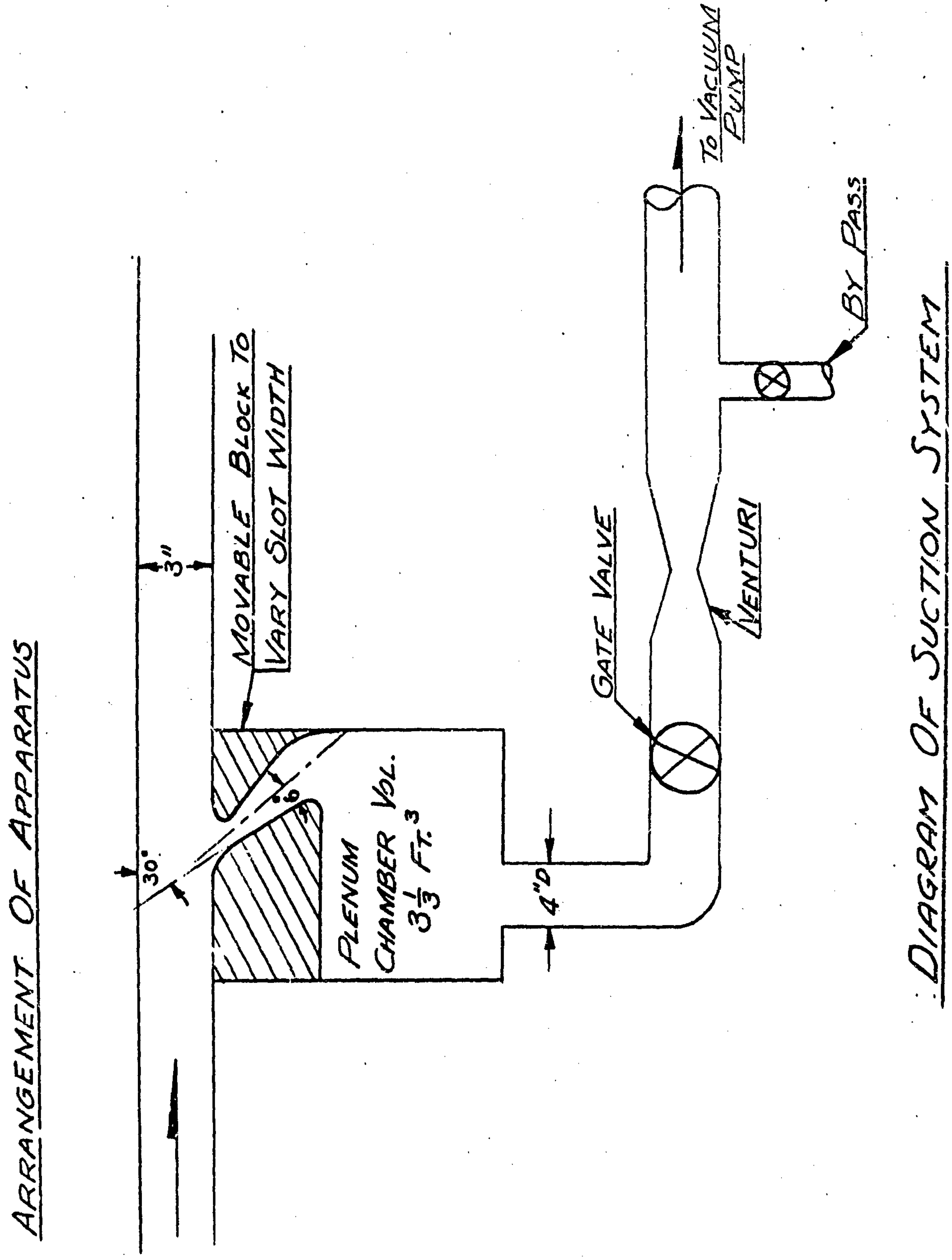
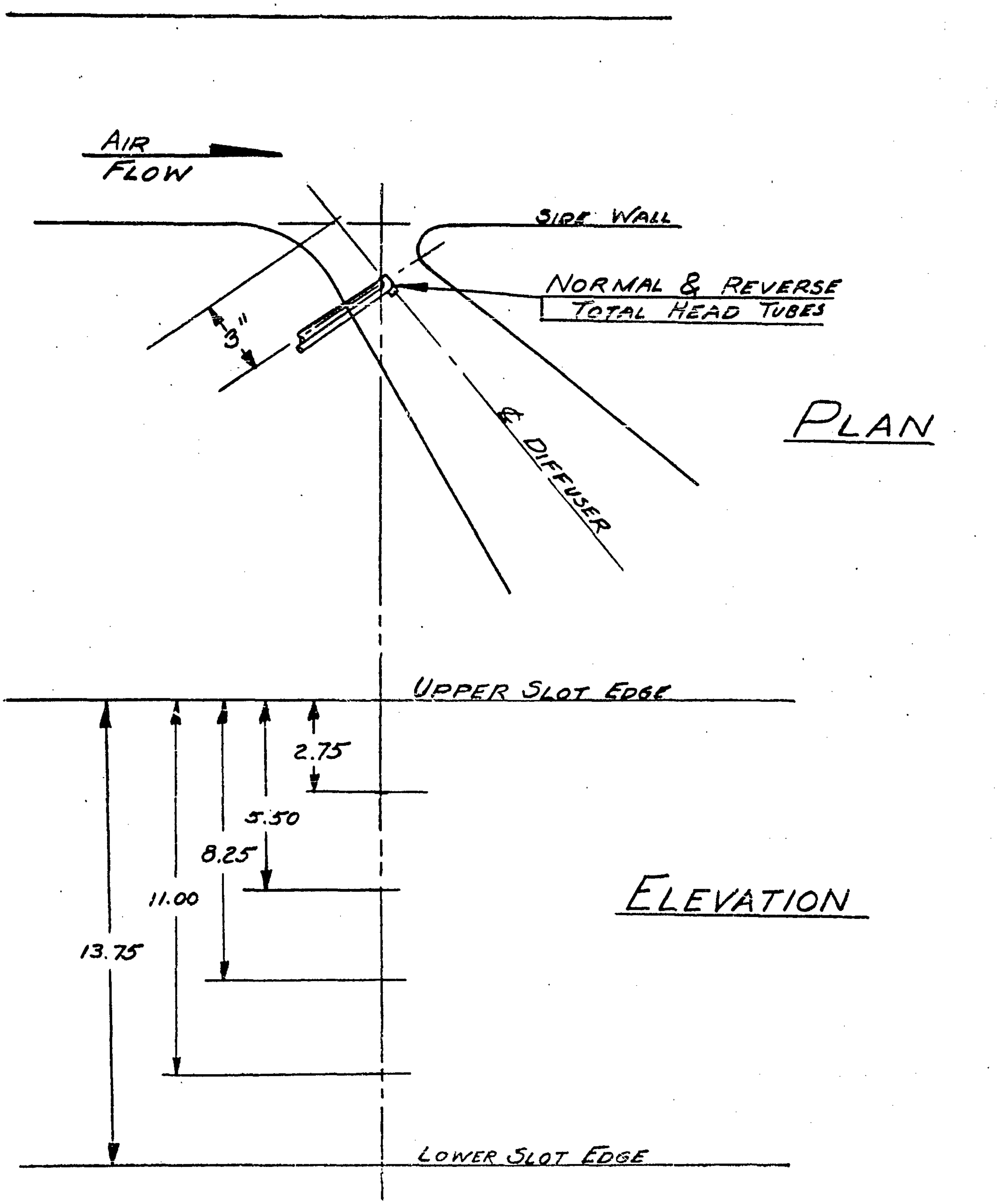

ARRANGEMENT OF APPARATUS
MOVABLE BLOCK TO VARY SLOT WIDTH
3"
30°
.6
PLENUM CHAMBER VOL. $3\frac{1}{3}$ FT.^{3}
4"D
GATE VALVE
VENTURI
BY PASS
TO VACUUM PUMP
DIAGRAM OF SUCTION SYSTEM

FIG. 4

LOCATION OF TOTAL-HEAD PROBES ACROSS DIFFUSER



DETAILS OF DIFFUSER THROTTLING VALVE

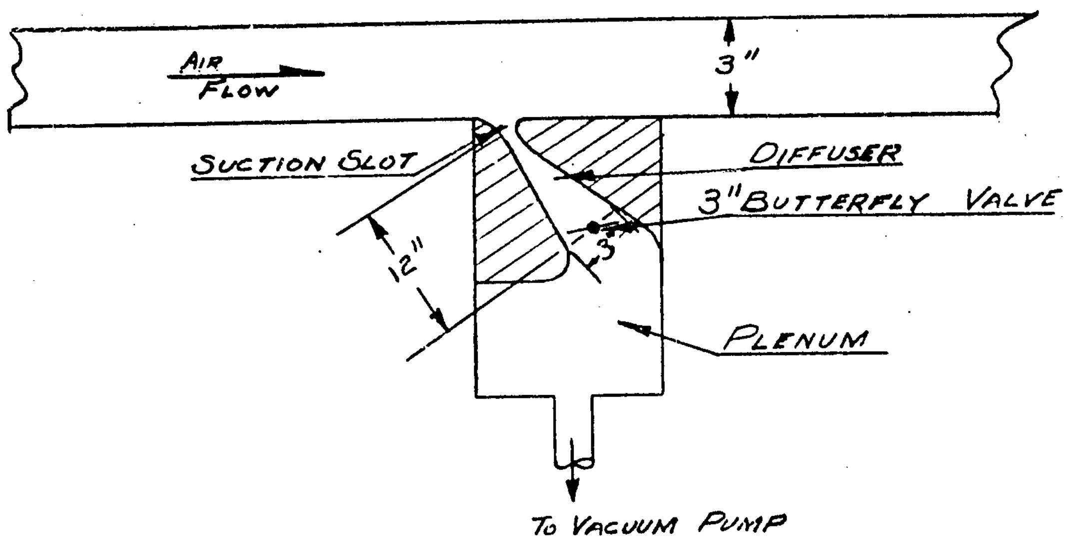
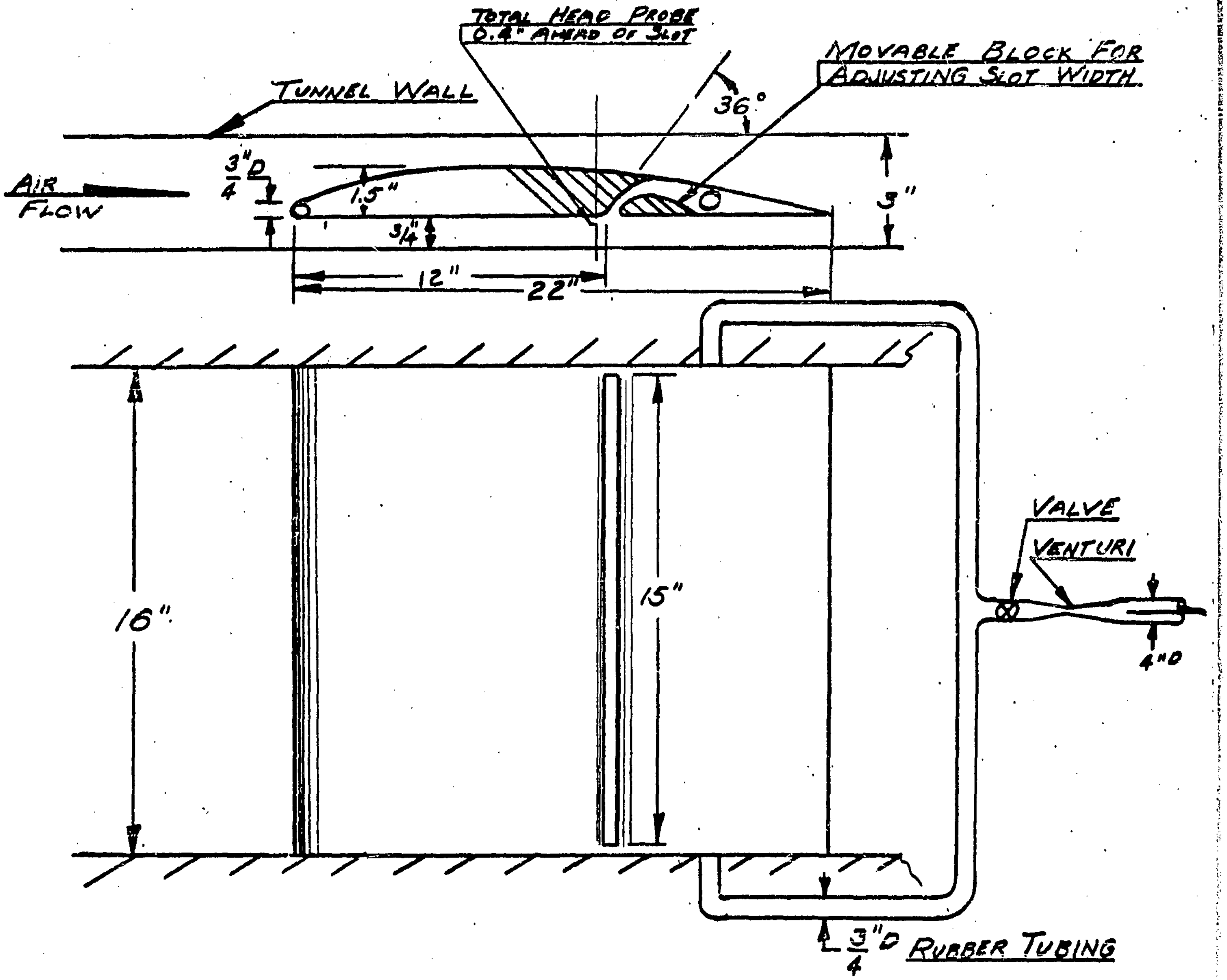


FIG. 6

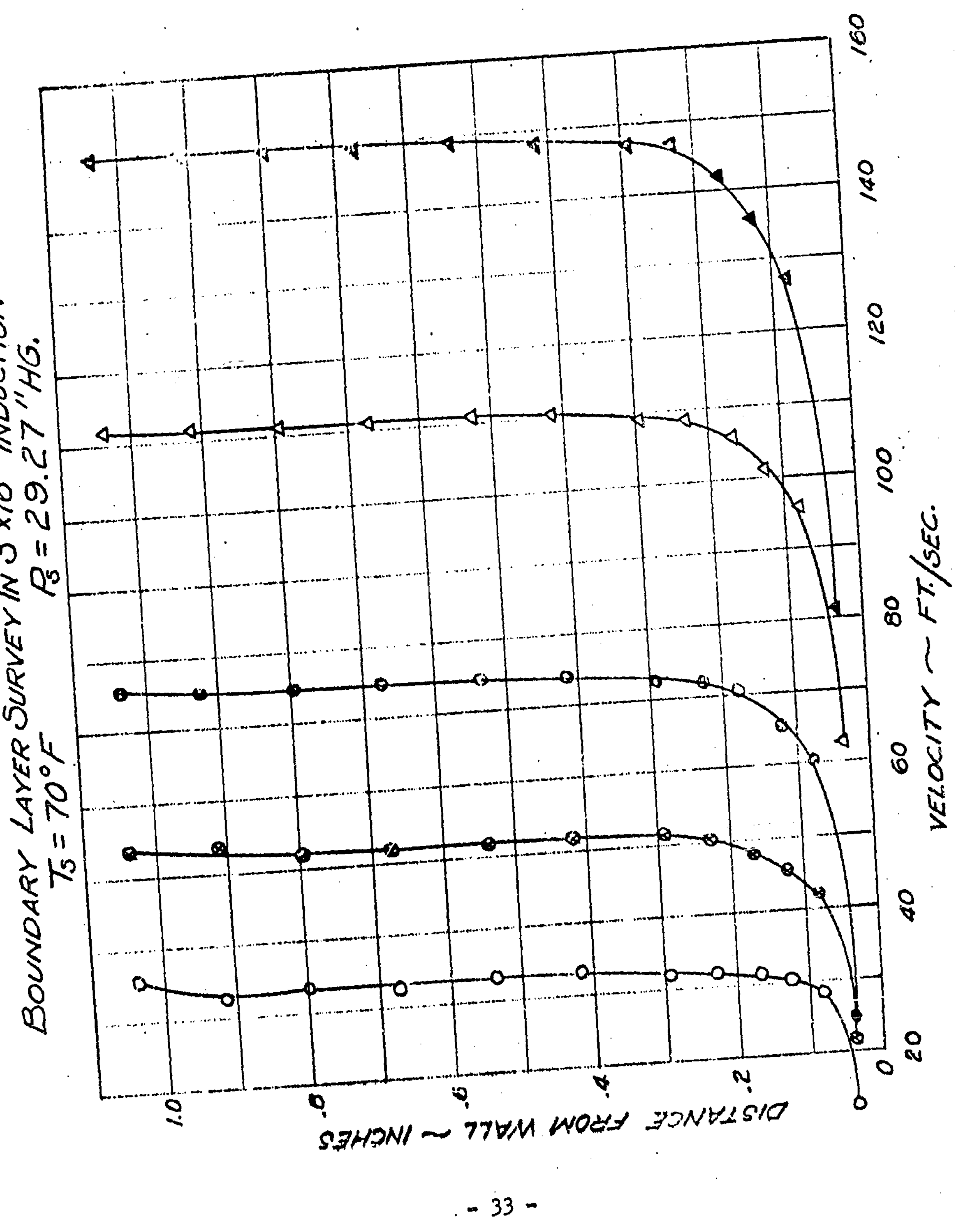
GENERAL ARRANGEMENT OF WIND TUNNEL AND AIRFOIL MODEL FOR AIRFOIL BOUNDARY-LAYER SUCTION



NOTE: SUCTION DIFFUSER SAME LINES AS LARGER MODEL
PLENUM VOLUME ≈ 0.045 FT.^{3}

Fig. 7

WALL BOUNDARY-LAYER PROFILES



DISTRIBUTION OF VELOCITIES ALONG SPAN OF SUCTION SLOT

FIG. 8

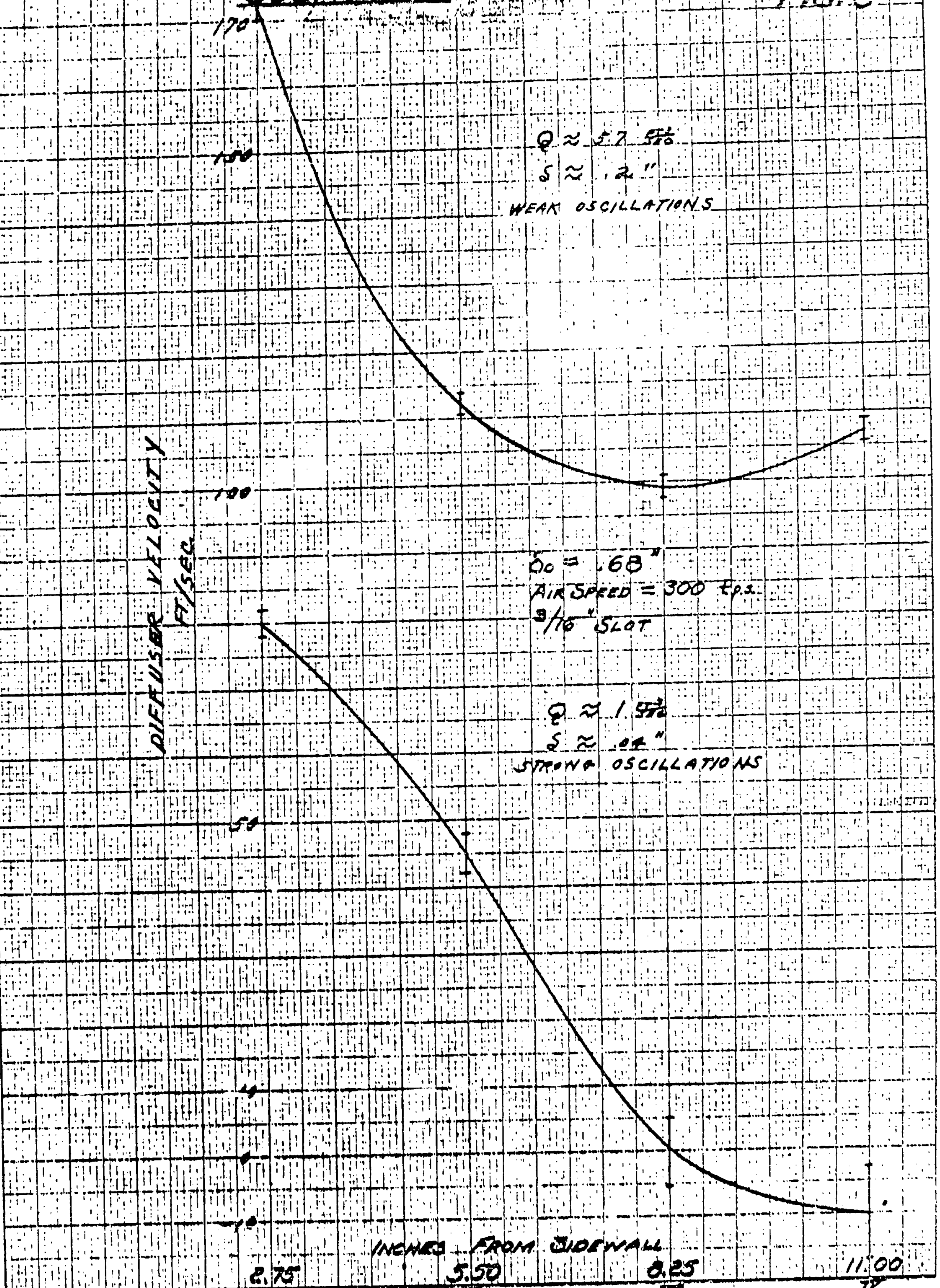


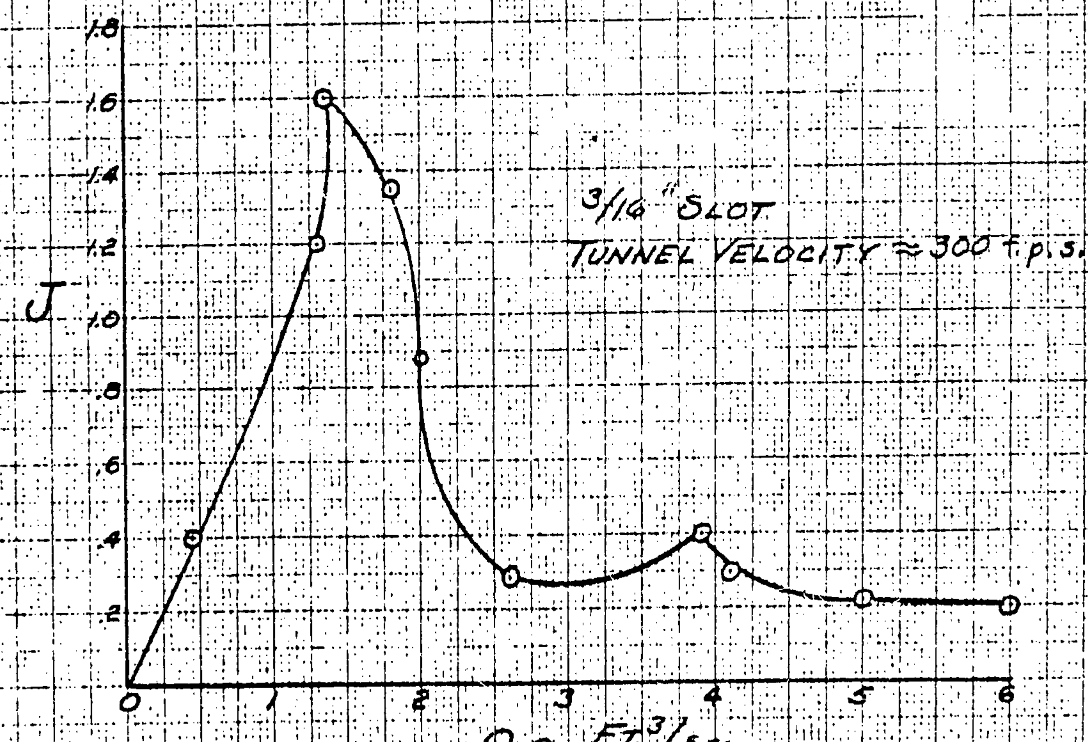
FIG. 9

FLOW ASYMMETRY INDEX VERSUS SUCTION FLOW QUANTITY

$$\text{ASYMMETRY INDEX, } J = \frac{u_{MAX.} - u_{MIN.}}{Q/A}$$

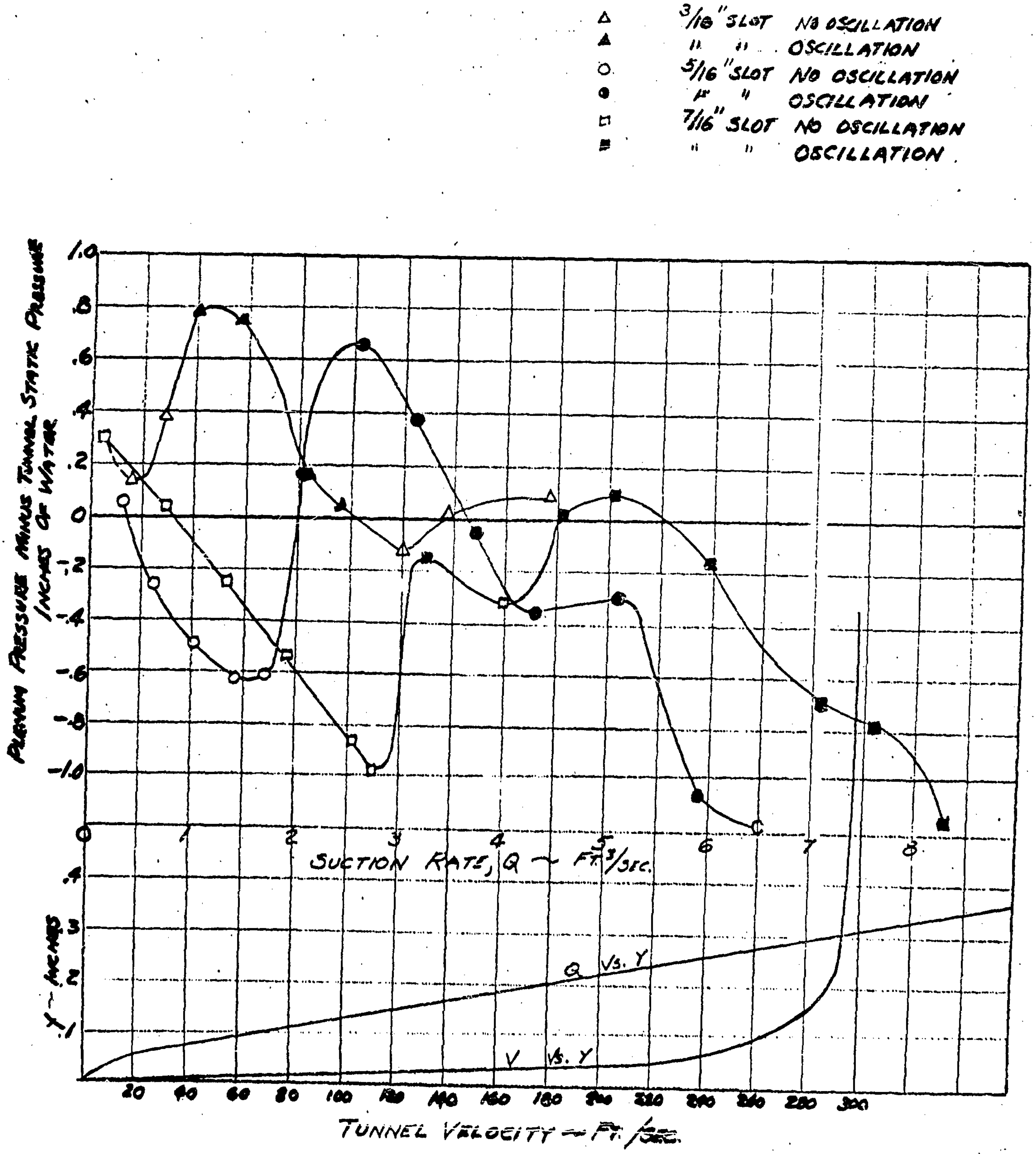
Q = VOLUME FLOW REMOVED BY SUCTION

A = AREA OF SLOT



PLENUM CHAMBER PRESSURE & DYNAMIC STABILITY
VS. SUCTION FLOW QUANTITY

FIG. 10



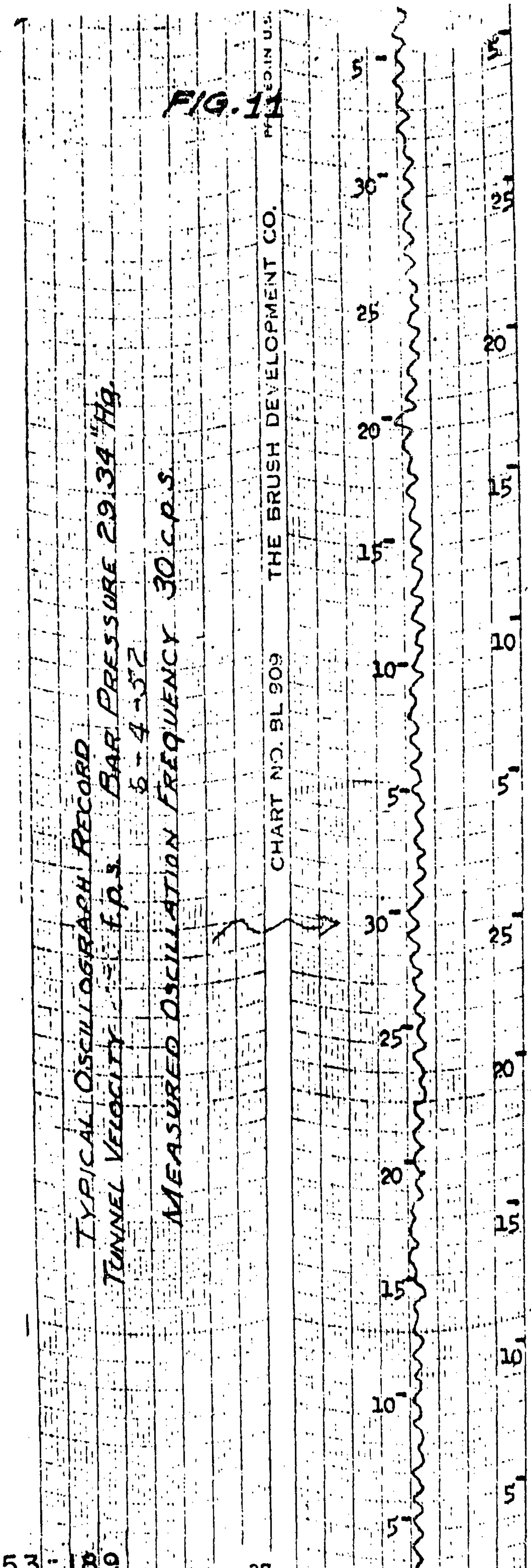
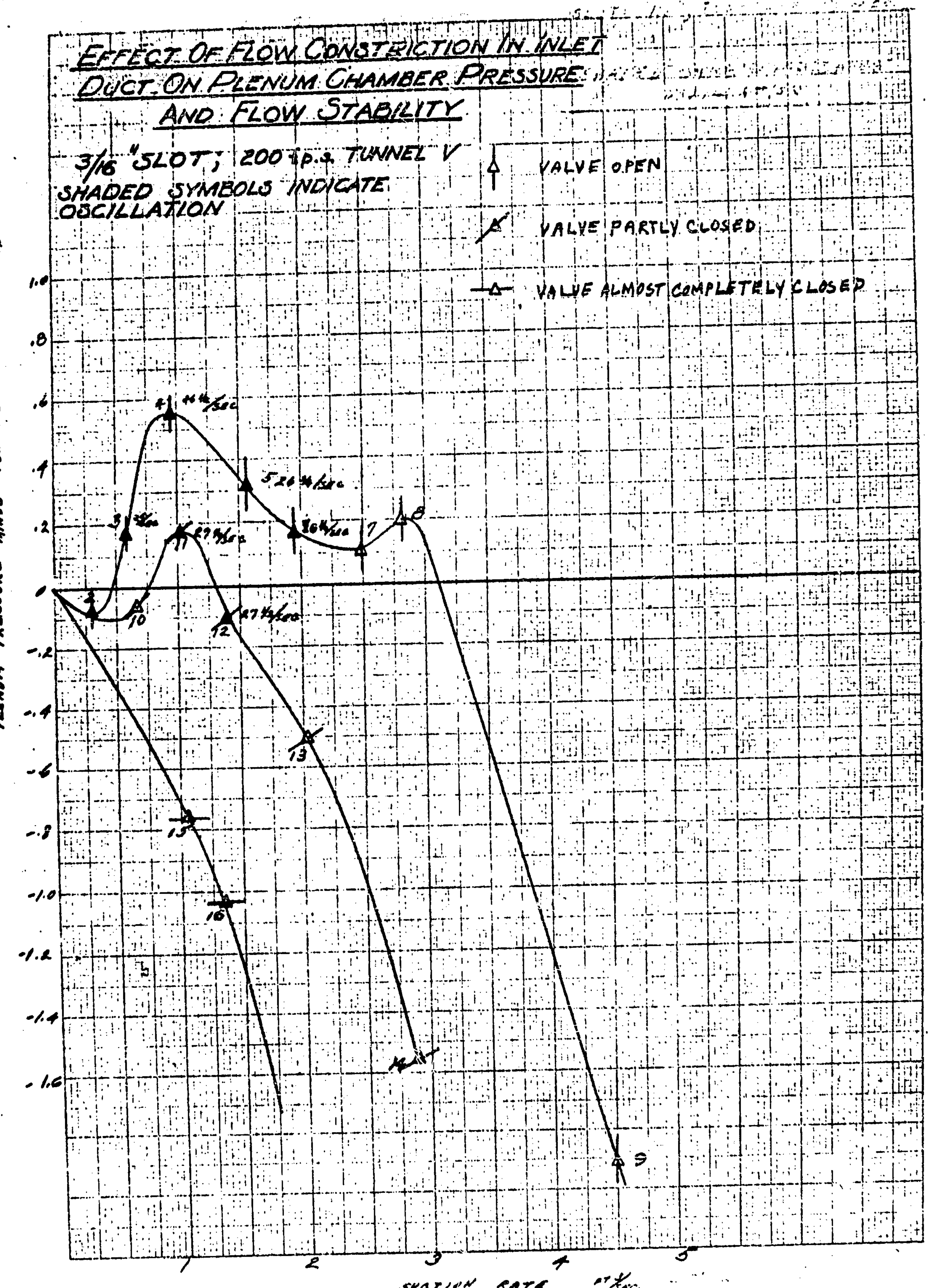

TYPICAL OSCILLOGRAPH RECORD
TUNNEL VELOCITY f.p.s. BAR PRESSURE 29.34" Hg.
6-4-52
MEASURED OSCILLATION FREQUENCY 30 c.p.s.
CHART NO. BL 909 THE BRUSH DEVELOPMENT CO.
PRINTED IN U.S.

FIG. 11


EFFECT OF FLOW CONSTRICTION IN INLET DUCT ON PLENUM CHAMBER PRESSURE AND FLOW STABILITY
3/16" SLOT; 200 f.p.s. TUNNEL V
SHADED SYMBOLS INDICATE OSCILLATION
VALVE OPEN
VALVE PARTLY CLOSED
VALUE ALMOST COMPLETELY CLOSED
PLENUM PRESSURE MINUS TUNNEL STATIC PRESSURE (INCHES OF WATER)
SUCTION RATE ft³/sec
1.0
.8
.6
.4
.2
0
-.2
-.4
-.6
-.8
-1.0
-1.2
-1.4
-1.6
1
2
3
4
5

EFFECT OF FLOW SPLITTERS IN INLET DUCT ON PLENUM CHAMBER PRESSURE & FLOW STABILITY

FIG. 13